

## Polygons

### Vocabulary:

**Polygon:** A shape with 3 or more sides made from segments that only intersect at their endpoints.

**Convex Polygon:** A polygon whose diagonals are only interior.

**Concave Polygon:** A polygon that has at least 1 diagonal that is exterior.

**Regular Polygon:** A polygon with all sides congruent and all angles congruent.

Cross out the shapes that are **NOT** polygons. Write *Concave* or *Convex* on the ones that are polygons.

Convex  
~~Concave~~



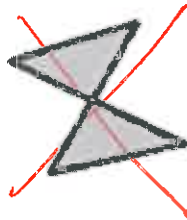
Concave



Concave



Convex



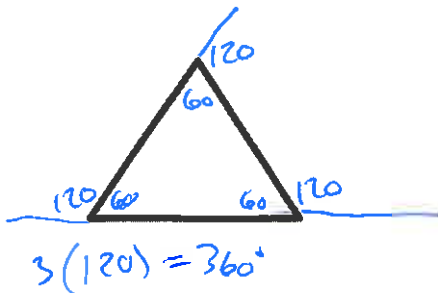
Convex



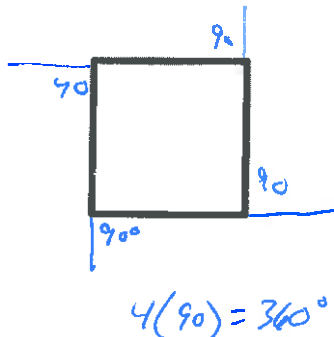
### Polygon Exterior Angle Sum

Draw the exterior angles for each polygon and find their sum.

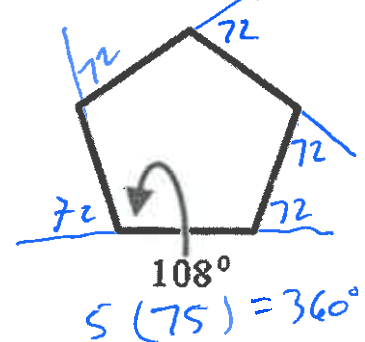
Equilateral Triangle



Square



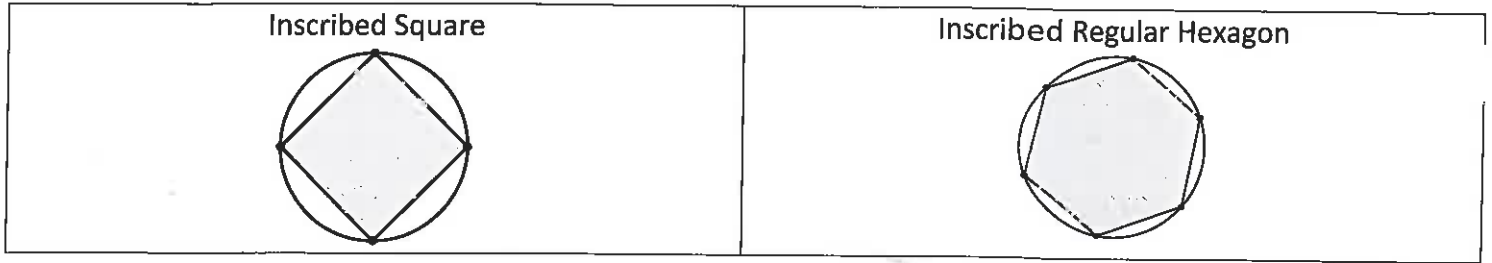
Regular Pentagon



What is the sum of the exterior angles of any polygon?

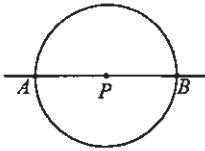
The ext.  $\angle$ 's always sum to  $360^\circ$

**Inscribed Polygon:** A polygon drawn inside a circle such that each vertex lies on the circle.

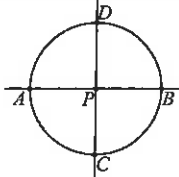


**Construct an Inscribed Square inside Circle P:**

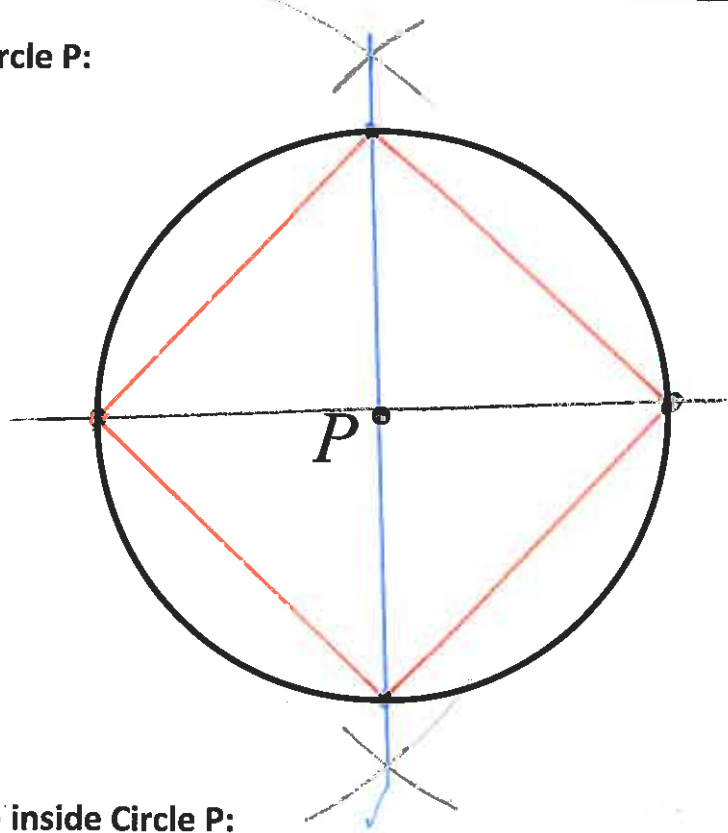
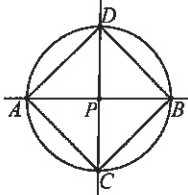
1. Construct diameter  $\overline{AB}$ .



2. Construct the  $\perp$  bisector of  $\overline{AB}$ .

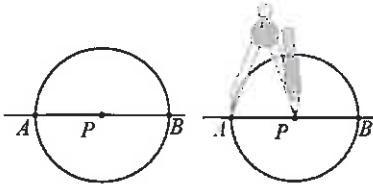


3. Construct the sides of the square.

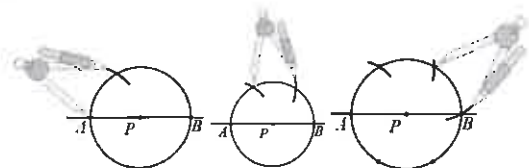


**Construct an Inscribed Regular Hexagon inside Circle P:**

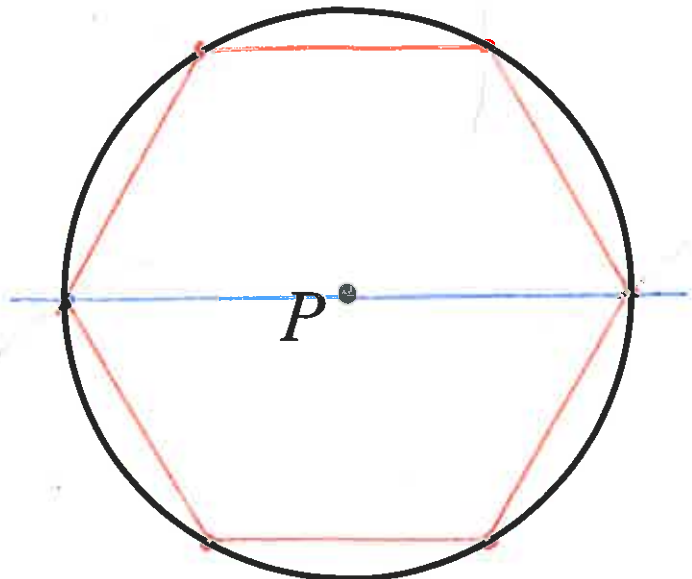
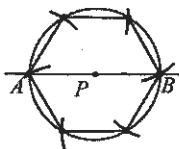
1. Construct diameter  $\overline{AB}$ . Measure  $\overline{AP}$ .



2. Construct arcs around the circle.



3. Complete the Regular Hexagon.

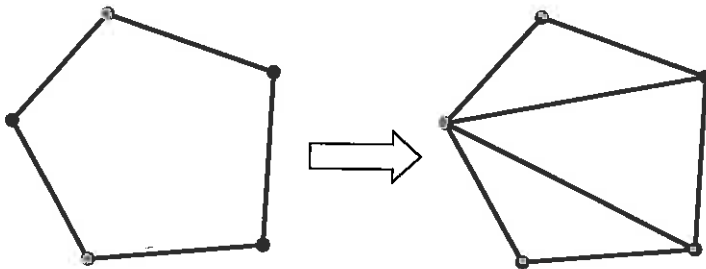


## Polygon - Method 1

**Directions:** This is **method 1** for finding the interior angle sum of a polygon. When you and your partner have completed this method, move on to Method 2.

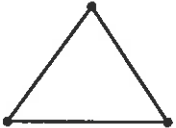
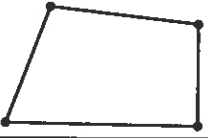
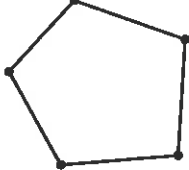
*Please, read and follow each step carefully!*

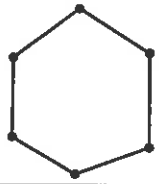
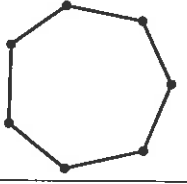
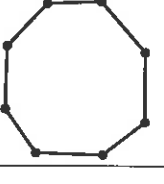
- The interior angle sum of a triangle = 180°.
- The interior angle sum of other polygons can be found by splitting the polygon into triangles. For example: A pentagon has 5 vertices. By choosing 1 vertex and connecting it to all other non-adjacent vertices, the pentagon is broken into 3 triangles:



So, the **interior angle sum** of a pentagon = (3 Triangles)(180° in each)  
= 540° total.

- Complete the table below using the same method described above.

Name	Picture	# of Sides	# of Triangles	Interior $\angle$ Sum
Triangle				
Quadrilateral				
Pentagon				

Hexagon				
Septagon (Heptagon)				
Octagon				

4. In the table, you will notice that there is a relationship between the **number of sides** of a polygon and the **number of triangles** formed. In the box below describe this relationship and then use your description to answer the questions that follow.

- a) How many triangles can be formed in a polygon with 12 sides? 10
- b) How many triangles can be formed in a polygon with 30 sides? 28
- c) How many triangles can be formed in a polygon with  $n$  sides?  $n-2$

5. In the table, the **interior angle sum** was found based on the **number of triangles** formed. In the box below, write a formula that gives the **interior angle sum** based on the **number of sides**. Use the variable  $n$  in your formula to represent the number of sides.

Interior angle sum =  $180(n-2)$

6. Test your formula by letting  $n = 5$  and  $n=8$ . Did you get the same values that are in the table? If not, revise your formula and try again.

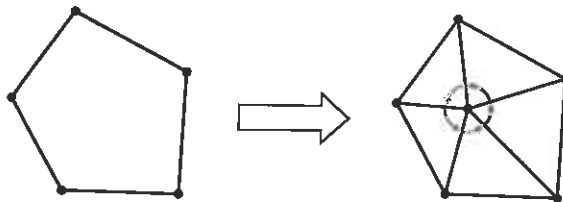
7. Move on to Method 2.

## Polygon - Method 2

Directions: This is **Method 2** for finding the interior angle sum of a polygon. When you and your partner have completed this method, move on to the Activity Summary.

*Please, read and follow each step carefully!*


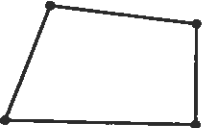
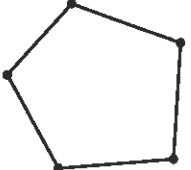
- The interior angle sum of a triangle = 180°.
- The interior angle sum of other polygons can be found by splitting the polygon into triangles. For example: A pentagon has 5 vertices. Draw a point anywhere inside the pentagon and connect it to each of the 5 vertices to form 5 triangles.

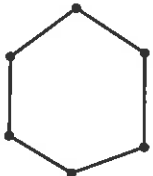

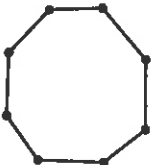


The 5 triangle vertices touching the interior point form a circle which is 360°.

$$\begin{aligned} \text{So, the interior angle sum of a pentagon} &= (5 \text{ triangles})(180^\circ \text{ each}) - 360^\circ \\ &= 540^\circ. \end{aligned}$$

- Complete the table below using the method described above.

Name	Picture	# of Sides	# of Triangles	Interior $\angle$ Sum
Triangle				
Quadrilateral				
Pentagon				

Hexagon				
Septagon (Heptagon)				
Octagon				

4. In the table, you will notice that there is a relationship between the **number of sides** of a polygon and the **number of triangles** formed. In the box below describe this relationship and then use your description to answer the questions that follow.

a) How many triangles are formed in a polygon with 12 sides? 12

b) How many triangles are formed in a polygon with 30 sides? 30

c) How many triangles are formed in a polygon with  $n$  sides?  $n$

5. In the table, the **interior angle sum** was found based on the **number of triangles** formed. In the box below, write a formula that gives the **interior angle sum** based on the **number of sides**. Use the variable  $n$  in your formula to represent the number of sides.

Interior angle sum =  $180n - 360$

6. Test your formula by letting  $n = 5$  and  $n = 8$ . Did you get the same values that are in the table? If not, revise your formula and try again.

7. Move on to the Activity Summary.

## Polygon - Activity Summary

**Directions:** This is the **Activity Summary** for finding the interior angle sum of a polygon.

1. In the box below, briefly describe how to calculate the interior angle sum of a polygon using method 1.

2. In the box below, briefly describe how to calculate the interior angle sum of a polygon using method 2.

3. In the activity you created two formulas. Write the formulas in the boxes below.

Method 1 Formula

$$180(n-2)$$

Method 2 Formula

$$180n - 360$$

What does  $n$  represent in each of the formulas?

4. Although the formulas look different, are they really different? Explain in this space.

5. With your partner answer the following questions:

a) What is the interior angle sum of a 25-gon?

$$180(25-2) = 180(23) = \boxed{4140^\circ}$$

b) What is the measure of one angle of a *regular* 15-gon?

(Hint: *regular* = all sides  $\cong$  & all angles  $\cong$ .)

$$\text{int } \angle \text{ sum} = 180(15-2) = 180(13) = 2340^\circ$$

$$\perp \text{ int } \angle = \frac{2340}{15} = \boxed{156^\circ}$$

c) How many sides does a polygon have if the interior angle sum is 7020°?

$$\text{int } \angle \text{ sum} = 7020$$

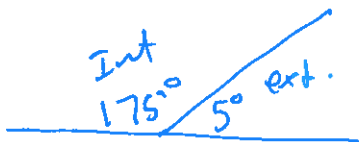
$$7020 = 180(n-2)$$

$$39 = n-2$$

$$\boxed{n = 41}$$

d) How many sides does a *regular* polygon have if one angle measures 175°?

(Hint: Try using exterior angles.)



$$\text{ext. } \angle \text{ sum} = 360^\circ$$

$$\perp \text{ ext} = \frac{360^\circ}{5^\circ} = 72 \text{ sides/angles.}$$

~~Side/angles~~

e)

