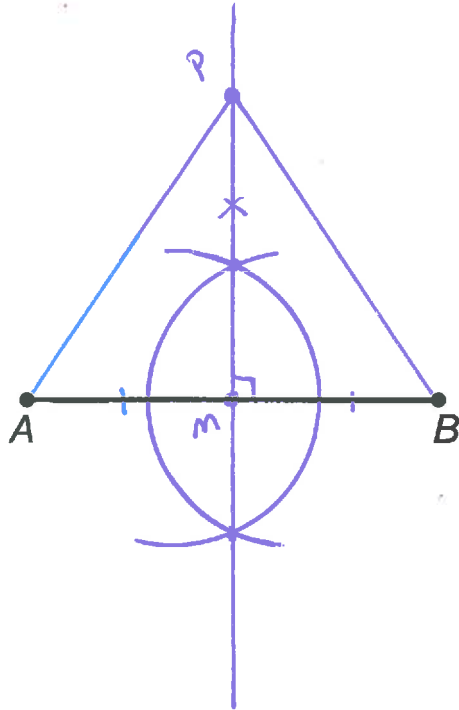


Perpendicular Bisectors & Circumcenters Angle Bisectors & Incenters

Perpendicular Bisector Theorem – Any point on the perpendicular bisector of a segment is **equidistant** to the endpoints of the segment.



1. Let's justify this theorem using a compass and straight edge:

a. Construct the perpendicular bisector of \overline{AB} and label the point of intersection M.

b. Place a point anywhere on the bisector and label it P. What does the **Perpendicular Bisector Theorem** claim is true in your picture? (*Write a congruency statement*).

$$\overline{PA} \cong \overline{PB}$$

c. Draw $\triangle APB$. Notice that \overline{AP} and \overline{BP} are corresponding parts? What triangle congruency postulate would be used to show $\triangle APM \cong \triangle BPM$?

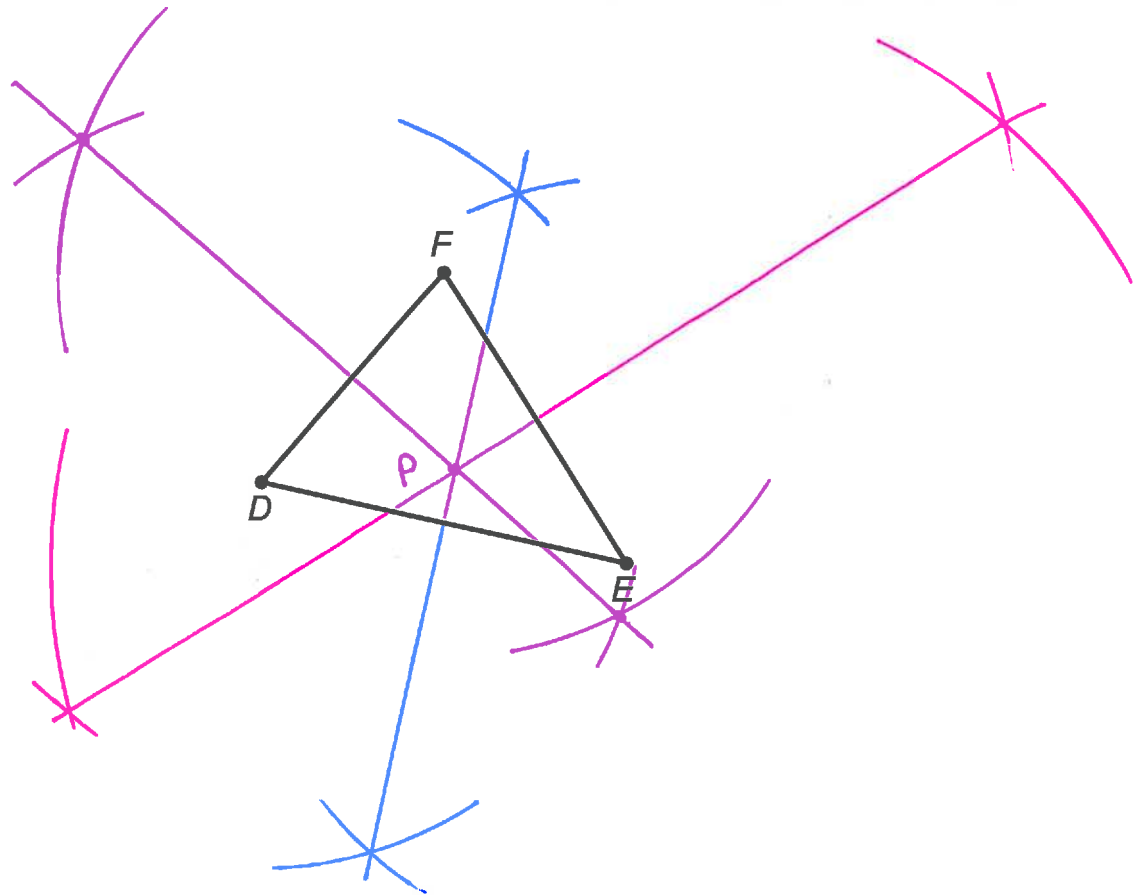
$$\triangle APM \cong \triangle BPM \text{ SAS.}$$

$$\text{So, } \overline{PA} \cong \overline{PB} \text{ by CPCTC.}$$

Point of Concurrence – The point where 3 or more lines intersect.

Circumcenter – The *point of concurrence* of the 3 **Perpendicular Bisectors** of a triangle.

2. Construct the perpendicular bisectors for each side of the triangle. Label the **Circumcenter P**.
(Be careful and precise or they may not intersect).



The Circumcenter Theorem – The **Circumcenter** is equidistant to the vertices of a triangle.

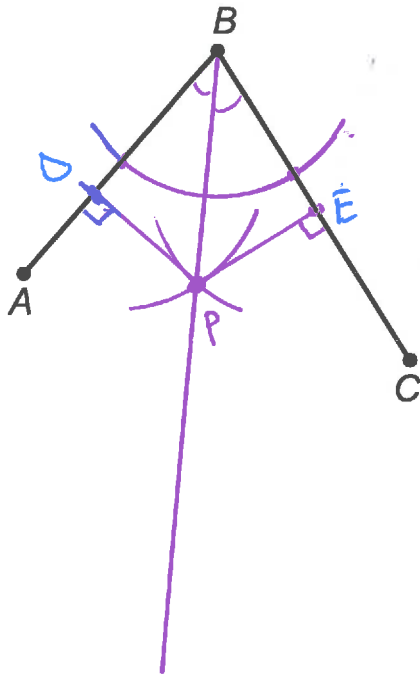
- 3a. What does the **Circumcenter Theorem** claim is true in your picture above? (Write a congruency statement)

$$\overline{PD} \cong \overline{PF} \cong \overline{PE}$$

- b. Explain how the **Perpendicular Bisector Theorem** implies that the **Circumcenter Theorem** must be true?

Since P is on the \perp bisector of each side, it must be equidistant to the endpoints of each side. The endpoints of each side are the vertices of the Δ .

Angle Bisector Theorem – Any point on the angle bisector of an angle is **equidistant** to the sides of the angle.



4. Let's justify this theorem using a compass and straight edge:

a. Construct the angle bisector of $\angle ABC$.

b. Place a point anywhere on the bisector, label it P.

c. The distance from a point to a line is always measured at a right angle. Using a protractor, place point D on \overline{BA} such that $\overline{BA} \perp \overline{PD}$. Also, place point E on \overline{BC} such that $\overline{BC} \perp \overline{PE}$.

d. What does the **Angle Bisector Theorem** claim is true in your picture? (Write a congruency statement)

$$\overline{PD} \cong \overline{PE}$$

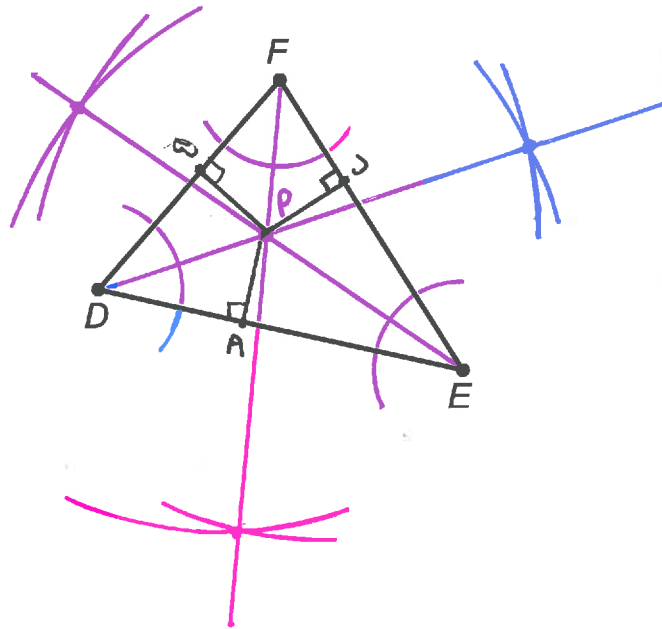
c. Notice that \overline{PD} and \overline{PE} are corresponding parts? What triangle congruency postulate would be used to show $\triangle BPD \cong \triangle BPE$?

$$\triangle BPD \cong \triangle BPE \text{ by AAS.}$$

$$\text{So, } \overline{PD} \cong \overline{PE} \text{ by CPCTC.}$$

Incenter – The point of concurrence of the 3 Angle Bisectors of a triangle.

5. Construct the angle bisectors for each angle of the triangle. Label the **Incenter P**.
(Be careful and precise or they may not intersect).



The Incenter Theorem – The Incenter is equidistant to the sides of a triangle.

- 6a. What does the **Incenter Theorem** claim is true in your picture above? (Write a congruency statement. Remember, the distance from a point to a line must be measured at a right angle.)

$$\overline{PB} \cong \overline{PA} \cong \overline{PC}$$

where: $\overline{PA} \perp \overline{DE}$
 $\overline{PB} \perp \overline{DF}$
and $\overline{PC} \perp \overline{FE}$

- b. Explain how the **Angle Bisector Theorem** implies that the **Incenter Theorem** must be true?

Since P is on the angle bisector of each angle of the Δ ,
then P must be equidistant to the sides of the angles.
The sides of the angles are also the sides of the Δ .