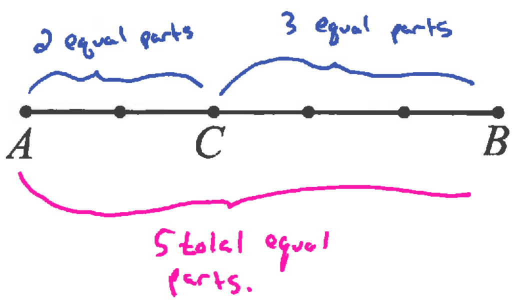
Geometry R – Mr. Bo
Unit 8 – Day 1 Notes

Name Notes.
Date____________________

Ratios and Proportions in Triangles

Ratio – A ratio is a comparison of the relative sizes of two quantities.

1.



a. A Ratio can express a "*Part to Part*" comparison. Describe a "Part to Part" ratio using $\overline{ACB}$.

ratio of AC to CB is 2:3

b. A Ratio can also express a "*Part to Whole*" comparison. Describe a "Part to Whole" ratio using $\overline{ACB}$.

ratio of AC to AB is 2:5

2. The angles of a triangle are in the ratio of 4:3:2.

a. Is this a "Part to Part" or "Part to Whole" ratio? How do you know?

Part to part because the ratio describes the relative sizes of the 3 angles (the "parts") to each other and not to the whole (180°).

b. Determine the measure of the largest angle, and show how you arrived at your answer.

4+3+2=9 Equal parts

All 3 ∠'s add to 180° ← the whole.

$\frac{180}{9} = 20$

Biggest angle $= 4(20) = 80°$

Here's how using Algebra:

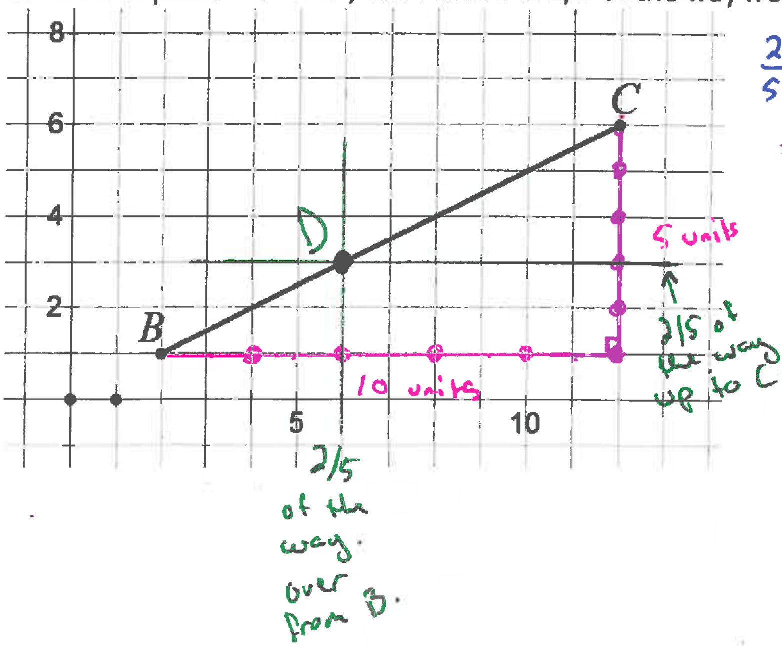
$4x+3x+2x=180$

$9x=180$

$x=20$

Largest ∠ $= 4x = 4(20) = 80°$

3. Find the point D on $\overline{BC}$, such that D is 2/5 of the way from B to C. Show how you arrived at your answer.



$\frac{2}{5}$ is a part to whole ratio.

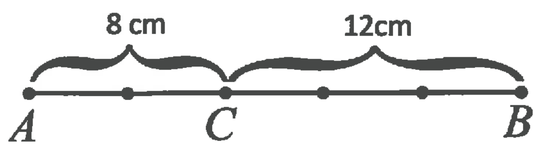
Draw a rt. △ and ÷ its legs into 5 equal parts.

$\frac{10}{5} = 2$ units and $\frac{5}{5} = 1$ unit.

D is the pt (6, 3).

Proportion: When two ratios are expressed as equal.

4.



In Example 1, you found the ratio of AC to CB to be 2:3. Suppose AC = 8cm and CB = 12cm. How is the ratio of AC to CB still 2:3?

$\frac{8}{12} = \frac{2(4)}{3(4)} = \frac{2}{3}$ by reducing we see the fractions are =.

Since the ratios "2:3" and "8:12" are equal, they can be written as a proportion:

extremes

$$2:3 = 8:12$$

means

Proportion Postulate: In a proportion, the product of the mean values equals the product of the **extreme values**.

The **mean values** of a proportion are the "**Inner**" terms.

The **extreme** values of a proportion are the "**outer**" terms.

5. Identify the mean and extreme values in the above proportion and show that their products are equal.

means: $3(8) = 24$ extremes: $2(12) = 24$ ✓

6. Ratios can also be expressed as fractions. Thus the proportion from question 4 can be written as

$$\frac{2}{3} = \frac{8}{12}$$

means

extremes.

When a proportion is written in this form, what is the more common name used to describe the *Proportion Postulate*? Cross multiplication.

7. The *Proportion Postulate* can be used to find missing quantities of a proportion. Solve each of the following proportions by applying the *Proportion Postulate*.

a. $x+11 : x = 12 : 16$

$16(x+11) = 12x$

$16x + 176 = 12x$

$176 = -4x$

$\boxed{x = -44}$

b. $\frac{5}{15} = \frac{a}{8+a}$

$15a = 5(8+a)$

$15a = 40 + 5a$

$10a = 40$

$\boxed{a = 4}$

c. $\frac{x-4}{2} = \frac{3x}{x+4}$

$(x-4)(x+4) = 2(3x)$

$x^2 - 4x + 4x - 16 = 6x$

$x^2 - 16 = 6x$

$x^2 - 6x - 16 = 0$

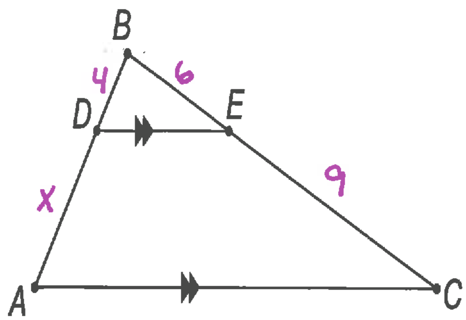
$(x-8)(x+2) = 0$

$x - 8 = 0$ | $x + 2 = 0$

$x = 8$ | $x = -2$

Proportions in Triangles

"Triangle Side Splitter" Theorem: A line that cuts across two sides of a triangle such that it is parallel to the third side, divides the lengths of the two sides ***proportionally***.



The phrase *"divides the lengths of the two sides proportionally"* means that a proportion can be established between the **ratios of the parts** (a *"Part to Part"* comparison).

8. Based on the theorem, which sides of ΔABC are being cut proportionally?

$\overline{AB}$ and $\overline{CB}$ since $\overline{DE} \parallel \overline{AC}$.

9. What proportion can be established from these two sides?

$$\frac{BD}{AD} = \frac{BE}{CE}$$

10. Suppose BE=6, EC=9, BD=4.

a. Find AD.

$\frac{4}{x} = \frac{6}{9}$

$6x = 4(9)$

$6x = 36$

$\boxed{x = 6}$ AD=6

b. Find AB.

$AB = AD + DB$ Seg. Addition.

$AB = 4 + 6$

$AB = 10$

c. Are $\frac{BD}{BA}$ and $\frac{BE}{BC}$ equal? How do you know?

$\frac{BD}{BA} = \frac{4}{10}$ $\frac{BE}{BC} = \frac{6}{15}$

$\frac{4}{10} = \frac{6}{15}$?

$4(15) = 6(10)$

$60 = 60$ ✓

they are = because the prod. of the means = prod. of the extremes.

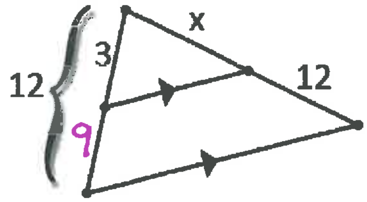
11. In addition to a *"Part to Part"* comparison, the phrase *"divides the lengths of the two sides proportionally"* also means that a *"Part to Whole"* comparison can be made, as seen in 10c above.

Using ΔABC, write the two proportions which represent *"Part to Whole"* comparisons.

$\frac{BD}{BA} = \frac{BE}{BC}$ and $\frac{AD}{BA} = \frac{CE}{BC}$

12. Solve for x, in each: *(Diagrams are not drawn to scale)*

a.

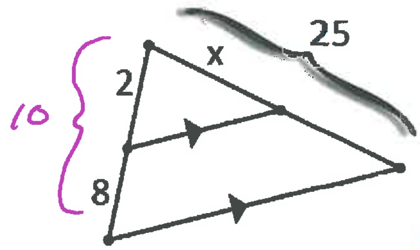


$\frac{3}{9} = \frac{x}{12}$ "part to part"

$9x = 36$

$\boxed{x = 4}$

b.



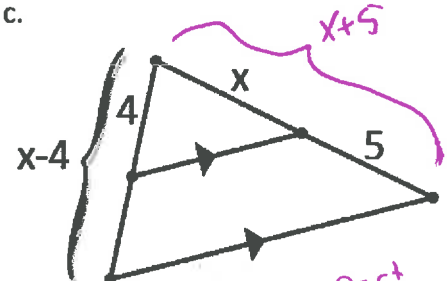
"part to whole"

$\frac{2}{10} = \frac{x}{25}$

$10x = 50$

$x = 5$

c.



$\frac{\text{part}}{\text{whole}}$

$\frac{4}{x-4} = \frac{x}{x+5}$

$4(x+5) = x(x-4)$

$4x + 20 = x^2 - 4x$

$20 = x^2 - 8x$

$0 = x^2 - 8x - 20$

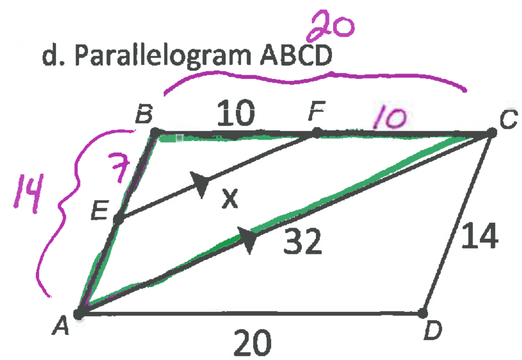
$0 = (x-10)(x+2)$

$x - 10 = 0$ | $x + 2 = 0$

$x = 10$ | $x = -2$

reject. (sides of $\triangle$s can't have neg. length)

d. Parallelogram ABCD



using $\triangle ABC$: $\overline{EF} \parallel \overline{AC}$ so.

$\frac{BE}{BA} = \frac{BF}{BC}$

$\frac{BE}{14} = \frac{10}{20}$

$20(BE) = 10(14)$

$BE = 7$

so, F is a midpt, E is a midpt } makes $\overline{EF}$ a midsegment.

So, $EF = \frac{1}{2}(AC)$

$= \frac{1}{2}(32)$

$= 16$