Geometry R – Mr. Bo
Unit 9 – Day 1 Notes

Name notes.
Date_______________

Pythagorean Theorem & Special Right Triangles

Reducing Radicals (Square Roots) to *Simplest Radical Form*

A square root is in ***Simplest Radical Form*** when:

1. All perfect squares have been removed from the radical.
2. All fractions have been removed from the radical.
3. No radicals remain in the denominator of a fraction.

1. Write each in simplest radical form:

a. $\sqrt{36}$

$= 6$

b. $\sqrt{72}$

$= \sqrt{36 \cdot 2}$

perfect square.

$= 6\sqrt{2}$

c. $5\sqrt{108}$

$= 5\sqrt{36 \cdot 3}$

$= 5 \cdot 6\sqrt{3}$

$= 30\sqrt{3}$

d. $\frac{2}{3}\sqrt{27}$

$= \frac{2}{3}\sqrt{9 \cdot 3}$

$= \frac{2}{\cancel{3}}(\cancel{3})\sqrt{3}$

$= 2\sqrt{3}$

e. $(3\sqrt{5})(2\sqrt{6})$

$= (3 \cdot 2)(\sqrt{5}\sqrt{6})$

$= 6\sqrt{30}$

no perfect squares go into 30.

f. $(5\sqrt{7})^2$

$= (5\sqrt{7})(5\sqrt{7})$

$= (5 \cdot 5)(\sqrt{7} \cdot \sqrt{7})$

$= 25\sqrt{49}$

perfect square.

$= 25 \cdot 7$

$= 175$

g. $\frac{\sqrt{80}}{\sqrt{125}}$

$= \frac{\sqrt{16 \cdot 5}}{\sqrt{25 \cdot 5}}$

$= \frac{4\cancel{\sqrt{5}}}{5\cancel{\sqrt{5}}}$

$\boxed{= 4/5}$

h. $\frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{2\sqrt{5}}{\sqrt{25}}$

$\boxed{= \frac{2\sqrt{5}}{5}}$

i. $\sqrt{\frac{2}{27}}$

$= \frac{\sqrt{2}}{\sqrt{9 \cdot 3}}$

$= \frac{\sqrt{2}}{3\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

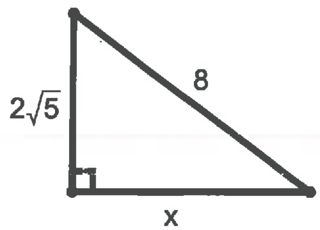
$= \frac{\sqrt{6}}{3 \cdot 3}$

$= \frac{\sqrt{6}}{9}$

Pythagorean Theorem: $a^2 + b^2 = c^2$

2. Solve for x in each.

a.



$x^2 + (2\sqrt{5})^2 = 8^2$

$x^2 + (4 \cdot 5) = 64$

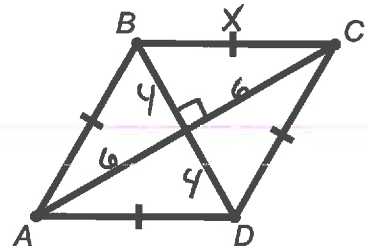
$x^2 + 20 = 64$

$x^2 = 44$

$x = \sqrt{44}$

$= 4\sqrt{11}$

b. Rhombus ABCD. BD=8, AC= 12. BC=x.



$4^2 + 6^2 = x^2$

$16 + 36 = x^2$

$52 = x^2$

$x = \sqrt{52}$

$= \sqrt{4 \cdot 13}$

$= 2\sqrt{13}$

Pythagorean Theorem Converse:

Acute Triangle	Right Triangle	Obtuse Triangle
$a^2+b^2 > c^2$	$a^2+b^2=c^2$	$a^2+b^2<c^2$

3. Determine if the triangle is Right, Acute, or Obtuse.

a. {6, 14, 18}

$6^2 + 14^2$

$= 36 + 196$

$= 232$

18^2

$= 324$

$a^2+b^2 < c^2$

obtuse Δ.

b. {6, $\sqrt{8}$, $2\sqrt{11}$}

$6^2 + (\sqrt{8})^2$

$= 36 + 8$

$= 44$

$(2\sqrt{11})^2$

$= 4 \cdot 11$

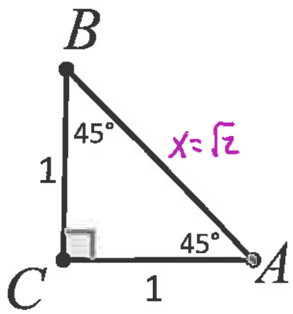
$= 44$

$a^2+b^2=c^2$

right Δ.

Special Right Triangles

45-45-90 Triangle (Half of a Square)



4. Right ΔABC is called a **45-45-90 triangle** because of the measures of its three angles. ΔABC is half of a square.

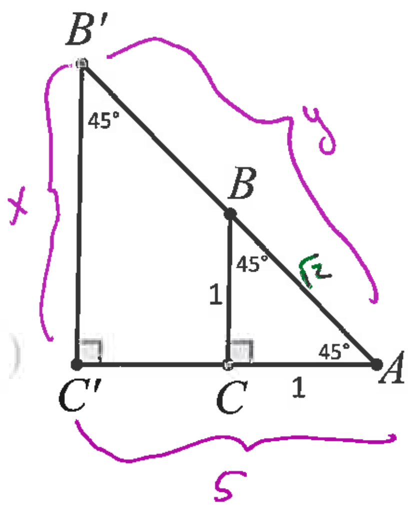
a. Suppose the legs of ΔABC are each 1 unit in length. Find the length of the hypotenuse.

$1^2+1^2=x^2$

$2=x^2$

$x=\sqrt{2}$

hypotenuse $AB=\sqrt{2}$



b. If ΔABC is dilated around center A, then $\Delta AB'C'$ is also a 45-45-90 triangle and $\Delta ABC \sim \Delta AB'C'$. Why?

Dilation preserves ∠ measure so, the Δ's are ~ by AA similarity.

c. Suppose $AC'=5$. Using the fact that the triangles are similar, find the lengths of the second leg and the hypotenuse of $\Delta AB'C'$. Show how you arrived at your answers.

x = second leg

$\frac{AC}{AC'}=\frac{BC}{B'C'}$

$\frac{1}{5}=\frac{1}{x}$

$\boxed{x=5}$

*Since the Δ's are ~ we can set up proportions. $\left(\frac{\text{Small}}{\text{Big}}=\frac{\text{Small}}{\text{Big}}\right)$

y = hypotenuse

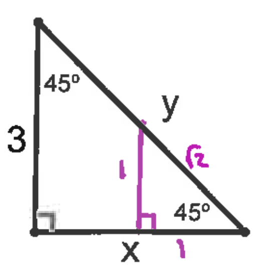
$\frac{AB}{AB'}=\frac{AC}{AC'}$

$\frac{\sqrt{2}}{y}=\frac{1}{5}$

$\boxed{y=5\sqrt{2}}$

5. Find x and y in **simplest radical** form.

a.



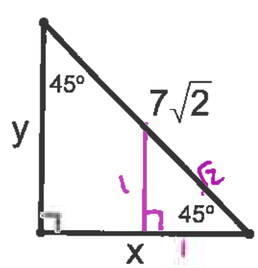
$\frac{1}{x}=\frac{1}{3}$

$\boxed{3=x}$

$\frac{\sqrt{2}}{y}=\frac{1}{3}$

$\boxed{3\sqrt{2}=y}$

b.



$\frac{1}{x}=\frac{\sqrt{2}}{7\sqrt{2}}$

$7\sqrt{2}=\sqrt{2}x$

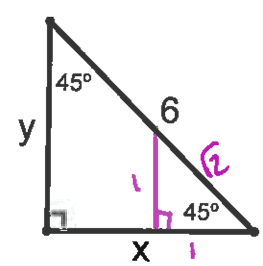
$\boxed{7=x}$

$\frac{1}{y}=\frac{\sqrt{2}}{7\sqrt{2}}$

$7\sqrt{2}=y\sqrt{2}$

$\boxed{y=7}$

c.



$\frac{1}{x}=\frac{\sqrt{2}}{6}$

$6=x\sqrt{2}$

$x=\frac{6}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$

$x=\frac{6\sqrt{2}}{2}$

$\boxed{x=3\sqrt{2}}$

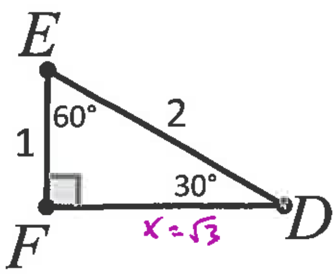
$\frac{1}{y}=\frac{\sqrt{2}}{6}$

$6=y\sqrt{2}$

$y=\frac{6}{\sqrt{2}}$

$\boxed{y=3\sqrt{2}}$

30-60-90 Triangle (Half of an Equilateral Triangle)

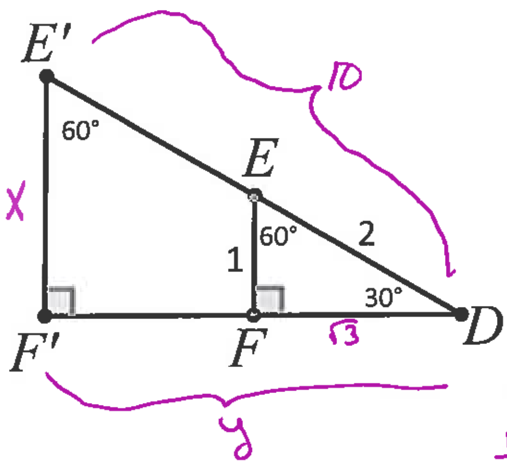


6. Right ΔDEF is called **a 30-60-90 triangle** because of the measures of its three angles. ΔDEF is half of an equilateral triangle.

a. Suppose the hypotenuse of ΔDEF is 2 units in length and the shorter leg is 1 unit (why?). Find the length of the longer leg.

$1^2 + x^2 = 2^2$
$1 + x^2 = 4$
$x^2 = 3$
$x = \sqrt{3}$

longer leg $DF = \sqrt{3}$



b. If ΔDEF is dilated around center D, then $\Delta DE'F'$ is also a 30-60-90 triangle and $\Delta DEF \sim \Delta DE'F'$. Why?

Dilation preserves ∠ measure so the Δ's are ~ by AA-sim.

c. Suppose $DE' = 10$. Using the fact that the triangles are similar, find the lengths of the legs of $\Delta DE'F'$. Show how you arrived at your answers.

x = short leg $\overline{E'F'}$

$\frac{1}{x} = \frac{2}{10}$

$2x = 10$

$\boxed{x = 5}$

Since Δ's are ~ we use a proportion.

y = long leg $\overline{DF'}$

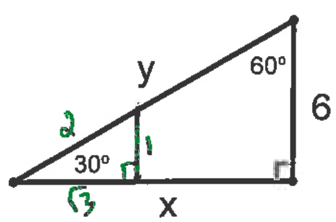
$\frac{\sqrt{3}}{y} = \frac{2}{10}$

$10\sqrt{3} = 2y$

$\boxed{y = 5\sqrt{3}}$

7. Find x and y, in **simplest radical** form.

a.



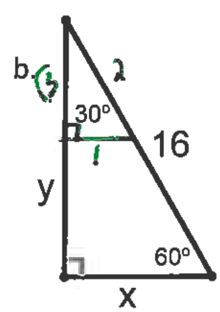
$\frac{1}{6} = \frac{\sqrt{3}}{x}$

$\boxed{x = 6\sqrt{3}}$

$\frac{1}{6} = \frac{2}{y}$

$\boxed{12 = y}$

b.



$\frac{1}{x} = \frac{2}{16}$

$2x = 16$

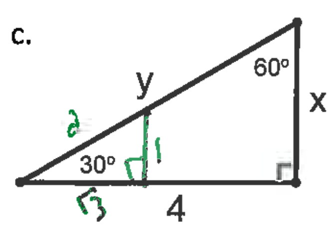
$\boxed{x = 8}$

$\frac{\sqrt{3}}{y} = \frac{2}{16}$

$16\sqrt{3} = 2y$

$\boxed{y = 8\sqrt{3}}$

c.



$\frac{1}{x} = \frac{\sqrt{3}}{4}$

$4 = x\sqrt{3}$

$x = \frac{4}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\boxed{x = \frac{4\sqrt{3}}{3}}$

$\frac{2}{y} = \frac{\sqrt{3}}{4}$

$8 = y\sqrt{3}$

$y = \frac{8}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\boxed{y = \frac{8\sqrt{3}}{3}}$