

Applications of Volume

Many real world objects can be modeled by basic 3D solids.

1. Identify the 3D solid that closely approximates the shape of each object.



Cylinder.



triangular prism.



cone.

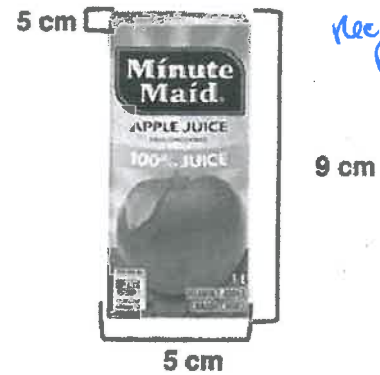
2a. Which apple juice container holds more juice?

Cylinder



$$\begin{aligned}
 V &= \pi r^2 h \\
 &= \pi (3)^2 (9) \\
 &= 254.5 \text{ cm}^3
 \end{aligned}$$

Rect. prism.



$$\begin{aligned}
 V &= B \cdot h \\
 &= (5)(5)(9) \\
 &= 225 \text{ cm}^3
 \end{aligned}$$

the can of juice holds more apple juice.

b. If apple juice costs \$2.80 per liter, how much does each juice container cost? (1 liter = 1000 cubic centimeters)

$$\frac{1 \text{ Liter}}{1000 \text{ cm}^3} = \frac{x \text{ liters}}{254.5 \text{ cm}^3}$$

$$x = \frac{254.5}{1000} = 0.2545 \text{ liters}$$

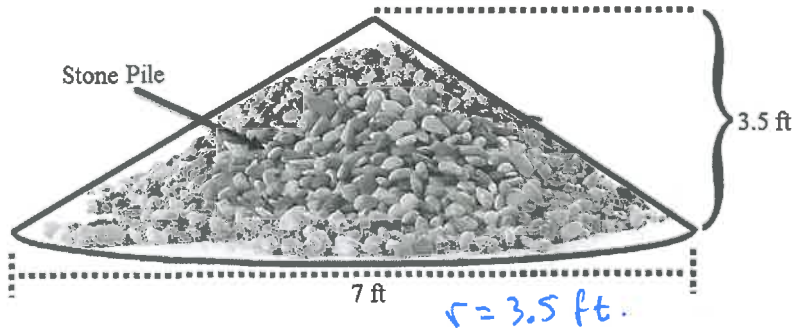
$$\text{Can costs } (\$2.80)(0.2545) = \$0.71$$

$$\frac{1 \text{ Liter}}{1000 \text{ cm}^3} = \frac{x \text{ liters}}{225 \text{ cm}^3}$$

$$x \text{ liters} = \frac{225}{1000} = 0.225 \text{ liters}$$

$$\text{Box costs } \$ (2.80)(0.225) = \$0.63$$

3. A landscaper has a pile of stone 3.5 feet high and 7 feet wide at its base. The pile can be approximated by a right cone, as seen in the picture.



- a. The term "yard" is commonly used to refer to a volume of material equal to one cubic yard. How many "yards" of stone does the landscaper have, to the nearest tenth?

$$V = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \pi (3.5)^2 (3.5)$$

$$= 44.9 \text{ ft}^3$$

$$\frac{1 \text{ yd}^3}{27 \text{ ft}^3} = \frac{x \text{ yds}^3}{44.9 \text{ ft}^3}$$

$$x = \frac{44.9}{27} = 1.7 \text{ yds}^3$$

the landscaper has about 1.7 "yards" of stone.

- b. The landscaper needs to transport the stone to a customer using his pickup truck. His truck can hold a maximum load of 1500 pounds. If the density of the stone is 1100 pounds per "yard", can he transport the stone using his truck in a single trip?

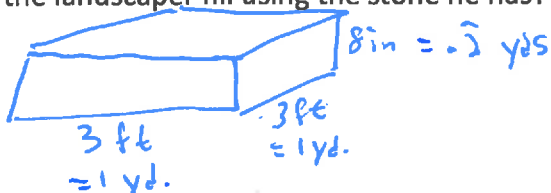
$$D = \frac{m}{v}$$

$$\frac{1100 \text{ lbs}}{1 \text{ yrd}^3} = \frac{x \text{ lbs}}{1.7 \text{ yds}^3}$$

$$x = (1.7)(1100) = 1,870 \text{ lbs}^2 \text{ of stone.}$$

the landscaper will need to make two trips to transport the stone to the customer.

- c. The landscaper will use the stone to fill a number of rectangular drainage ditches on the customer's property. If each drainage ditch is 3 feet wide by 3 feet long by 8 inches deep, how many drainage ditches can the landscaper fill using the stone he has?



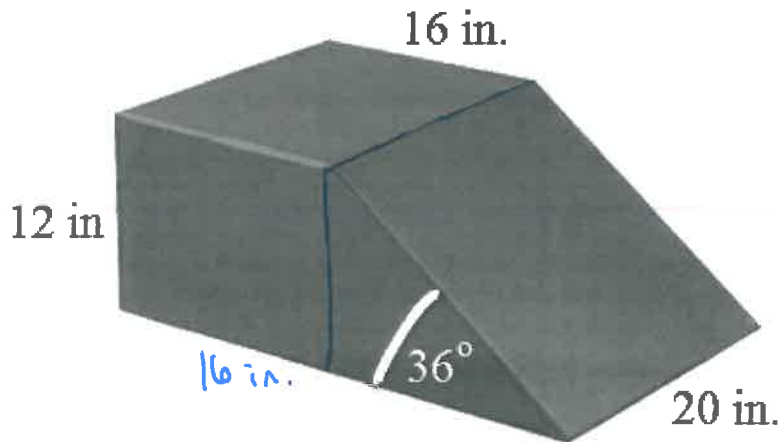
$$\text{Volume of 1 ditch} = (1)(1)(.2) \text{ yds}^3$$

$$= .2 \text{ yds}^3$$

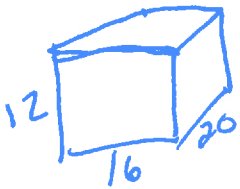
$$\frac{1.7 \text{ yds}^3 \text{ of stone}}{.2 \text{ yds}^3 \text{ for the ditches}} = 7.65$$

the landscaper can fill just over 7 and a half ditches.

4. An incline wedge is a special exercise mat used by physical therapists while rehabilitating patients. The incline wedge seen here is composed of a rectangular prism and a triangular prism.



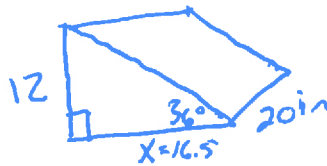
- a. The wedge is filled with special polymer foam. Determine the volume of the foam used to fill the wedge.



$$V = B \cdot h$$

$$= (16)(20)(12)$$

$$= 3,840 \text{ in}^3$$



$$V = \frac{1}{2} B \cdot h$$

$$= (99.1)(20)$$

$$= 1,982.0 \text{ in}^3$$

Base area:

$$\tan(36) = \frac{12}{x}$$

$$x = \frac{12}{\tan(36)} \approx 16.5$$

$$\text{Base area} = \frac{1}{2}(16.5)(12)$$

$$= 99.1 \text{ in}^2$$

$$\text{Total volume} = 3,840 + 1,982 = 5,822 \text{ in}^3$$

- b. The foam used to fill the wedge has a mass of 9 kilograms. Determine the **Density** (in lbs/in³) of the foam.
(Hint: The Density of an object is the ratio of its mass to its volume.)

1 kg = 2.2 pounds (on formula sheet)

$$\frac{1 \text{ kg}}{2.2 \text{ lbs}} = \frac{9 \text{ kg}}{x \text{ lbs}}$$

$$x \text{ lbs} = (2.2)(9) = 19.8 \text{ lbs}$$

mass of the foam is 19.8 lbs.

$$\text{Density} = \frac{\text{mass}}{\text{Volume}}$$

$$= \frac{19.8 \text{ lbs}}{5,822 \text{ in}^3}$$

$$= 0.003 \text{ lbs/in}^3$$

