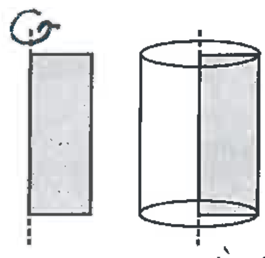
Geometry R – Mr. Bo
Unit 11 – Day 2 Notes

Name notes.
Date

Cavalieri's Principle – "Preservation of Volume"

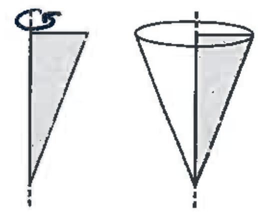
Identifying 3D Solids from a 2D cross section:

Solid by Revolution: A 3D solid can be formed by rotating a 2D figure around an axis.



cylinder.

Rectangle ⇒ cylinder.

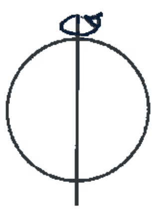


cone.

triangle ⇒ cone.

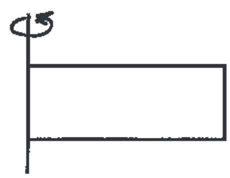
1. Name the 3D solid formed by rotating the 2D shape around the given axis.

a.



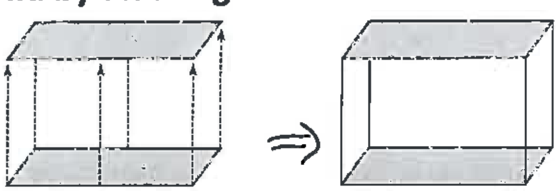
Sphere.

b.

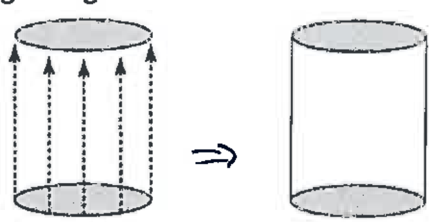


Cylinder.

Solid by Stacking: A 3D solid can be formed by stacking 2D figures.



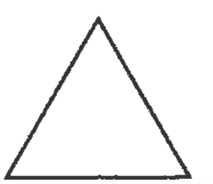
≅ Rectangles ⇒ rectangular Solid



≅ circles ⇒ cylinder.

2. Name the 3D solid formed by stacking 1000 (congruent) of the shapes given.

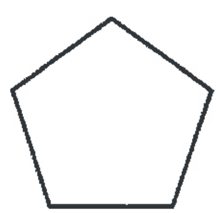
a.



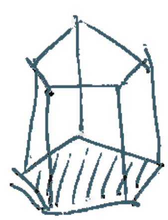
triangle ⇒

triangular prism.

b.

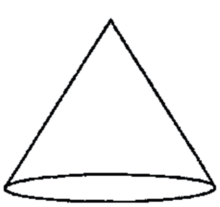


pentagon.



pentagonal prism

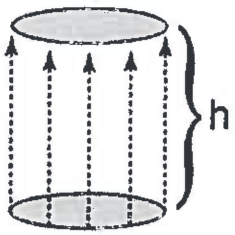
3. Describe how this 3D solid can be formed by stacking 2D shapes.

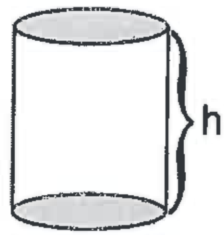


Stacking smaller and smaller circles to form the cone.

Cavalieri's Principle

4. Suppose 1000 circles were stacked to form a right cylinder of height *h*, as pictured.

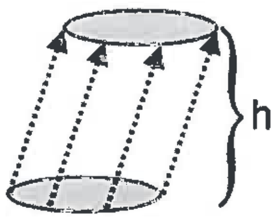


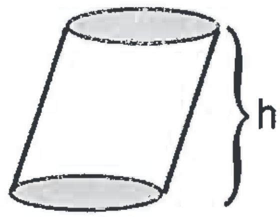


Describe how the volume of the cylinder can be derived from the area of the circular base.

Area of each circle = πr^2.
Volume of each is about πr^2 since the thickness of each circle is very small.
Since all the circles together have a height of h, the total volume would be $\pi r^2 h$.

5. Suppose, instead, the circles were stacked at an angle to form an ***Oblique Cylinder***.





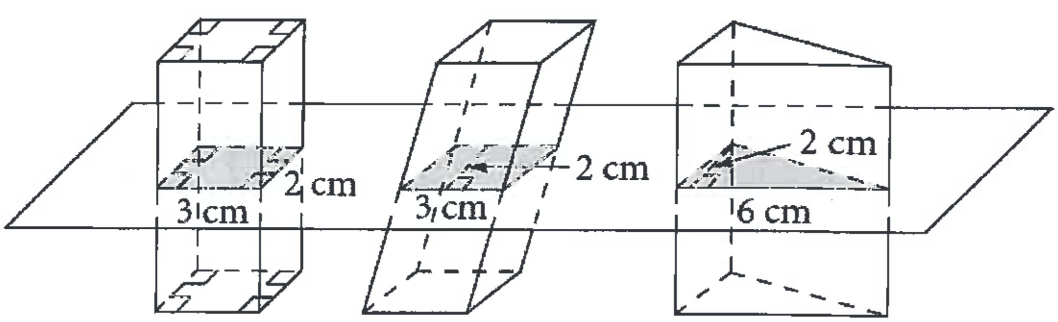
a. How does the height, h, of the oblique cylinder compare to that of the right cylinder in #4? Explain your reasoning.

the height would be the same, because the thickness ~~and~~ of each circle did not change and the # of circles used did not change.

b. How does the volume of the oblique cylinder compare to that of the right cylinder in #4? Explain.

the volumes would have to be equal. Since the # of circles used and the size of each circle did not change.

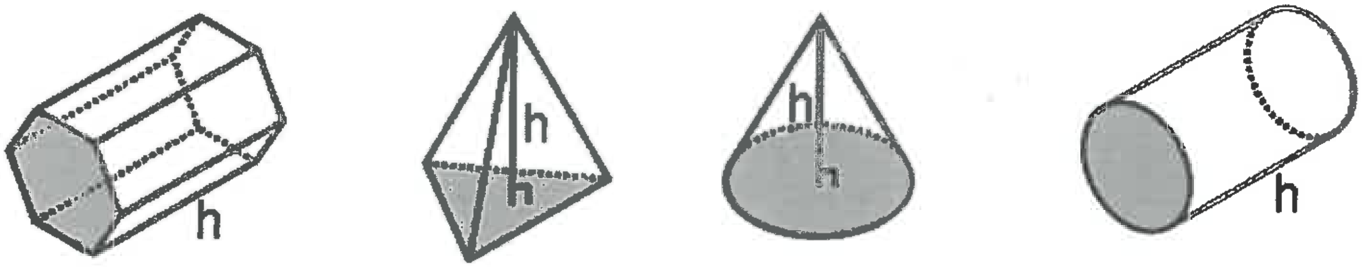
Cavalieri's Priniciple: If 2 solids have the **same height** and the **same cross sectional area at each level,** then the solids have the same volume.



Shape of the base does not matter.
As long as the cross section at each level have = area.

6. Each of the four solids has the same base area and height. Using Cavalier's Principle, explain which of the solids have the same volume.

a. b. c. d.



a and d have the same volume because their base areas are the same at each level.

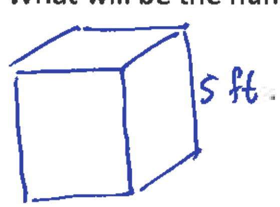
b and c. will have the same volume as long as the cross-sectional area remains the same at each level.

7. A carpenter made a storage container in the shape of a rectangular prism. It is 5 feet high and has a volume of 720 cubic feet. He wants to make a second container with the same height and volume as the first one, but in the shape of a triangular prism.

a. What does Cavalieri's Principle imply about the area of the base of the new container compared to the first container? Explain.

the area of the bases must be equal because the two shapes have the same volume and height.

b. What will be the number of square feet in the area of the base of the new container?



$V = 720 \text{ ft}^3$

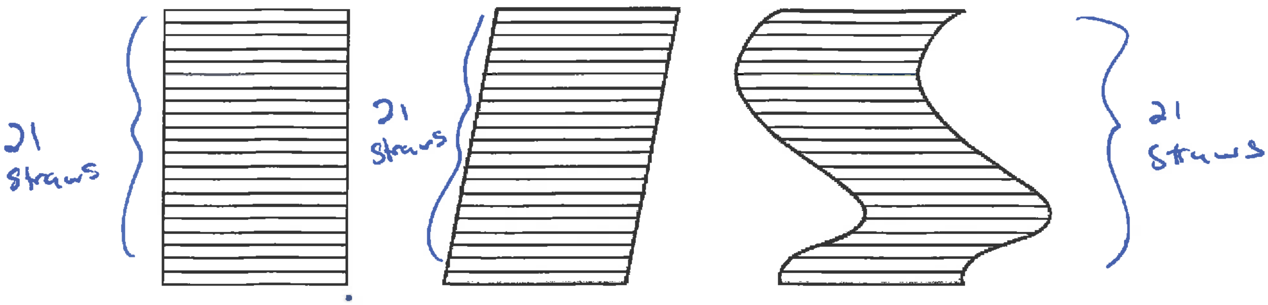
$V = B \cdot h$

$720 = B \cdot (5)$

$B = \frac{720}{5} = 144 \text{ ft}^2.$

the area of the base will be 144 ft^2.

8. The figures below can be covered by equal numbers of straws that are the same length.



a. Describe how Cavalieri's principle could be adapted to compare the areas of these figures.

Since each figured is covered by the same # of straws, the area of the figures ~~must~~ must be the same.

b. Suppose each straw is 15cm long and 5 mm wide. Determine the approximate area of each figure.

↳ 150mm.

Area of 1 straw = (150)(5) = 750 mm^2

total area of each figure = (750)(21) = 15,750 mm^2