Lesson 8: Adding and Subtracting Polynomials

Classwork

Exercise 1

a. How many quarters, nickels, and pennies are needed to make $\$1.13$?

b. Fill in the blanks:

$8{,}943 = \_\_\_\_\_ \times 1000 + \_\_\_\_\_ \times 100 + \_\_\_\_\_ \times 10 + \_\_\_\_\_ \times 1$

$= \_\_\_\_\_ \times 10^3 + \_\_\_\_\_ \times 10^2 + \_\_\_\_\_ \times 10 + \_\_\_\_\_ \times 1$

c. Fill in the blanks:

$8{,}943 = \_\_\_\_\_ \times 20^3 + \_\_\_\_\_ \times 20^2 + \_\_\_\_\_ \times 20 + \_\_\_\_\_ \times 1$

d. Fill in the blanks:

$113 = \_\_\_\_\_ \times 5^2 + \_\_\_\_\_ \times 5 + \_\_\_\_\_ \times 1$

Exercise 2

Now let's be as general as possible by not identifying which base we are in. Just call the base x.

Consider the expression $1 \cdot x^3 + 2 \cdot x^2 + 7 \cdot x + 3 \cdot 1$, or equivalently $x^3 + 2x^2 + 7x + 3$.

a. What is the value of this expression if $x = 10$?

b. What is the value of this expression if $x = 20$?

Exercise 3

a. When writing numbers in base 10, we only allow coefficients of 0 through 9. Why is that?

b. What is the value of $22x + 3$ when $x = 5$? How much money is 22 nickels and 3 pennies?

c. What number is represented by $4x^2 + 17x + 2$ if $x = 10$?

d. What number is represented by $4x^2 + 17x + 2$ if $x = -2$ or if $x = \frac{2}{3}$?

e. What number is represented by $-3x^2 + \sqrt{2}x + \frac{1}{2}$ when $x = \sqrt{2}$?

POLYNOMIAL EXPRESSION: A *polynomial expression* is either

1. A numerical expression or a variable symbol, or
2. The result of placing two previously generated polynomial expressions into the blanks of the addition operator (__+__) or the multiplication operator (__×__).

Exercise 4

Find each sum or difference by combining the parts that are alike.

a. $417 + 231 =$ _____ hundreds + _____ tens + _____ ones + _____ hundreds + _____ tens + _____ ones

$=$ _____ hundreds + _____ tens + _____ ones

b. $(4x^2 + x + 7) + (2x^2 + 3x + 1)$

c. $(3x^3 - x^2 + 8) - (x^3 + 5x^2 + 4x - 7)$

d. $3(x^3 + 8x) - 2(x^3 + 12)$

e. $(5 - t - t^2) + (9t + t^2)$

f. $(3p + 1) + 6(p - 8) - (p + 2)$

Lesson Summary

A *monomial* is a polynomial expression generated using only the multiplication operator (__×__). Thus, it does not contain + or − operators. *Monomials* are written with numerical factors multiplied together and variable or other symbols each occurring one time (using exponents to condense multiple instances of the same variable).

A *polynomial* is the sum (or difference) of monomials.

The *degree of a monomial* is the sum of the exponents of the variable symbols that appear in the monomial.

The *degree of a polynomial* is the degree of the monomial term with the highest degree.

Problem Set

1. Celina says that each of the following expressions is actually a binomial in disguise:
   i. $5abc - 2a^2 + 6abc$
   ii. $5x^3 \cdot 2x^2 - 10x^4 + 3x^5 + 3x \cdot (-2)x^4$
   iii. $(t+2)^2 - 4t$
   iv. $5(a-1) - 10(a-1) + 100(a-1)$
   v. $(2\pi r - \pi r^2)r - (2\pi r - \pi r^2) \cdot 2r$

   For example, she sees that the expression in (i) is algebraically equivalent to $11abc - 2a^2$, which is indeed a binomial. (She is happy to write this as $11abc + (-2)a^2$, if you prefer.)

   Is she right about the remaining four expressions?

2. Janie writes a polynomial expression using only one variable, x, with degree 3. Max writes a polynomial expression using only one variable, x, with degree 7.
   a. What can you determine about the degree of the sum of Janie's and Max's polynomials?
   b. What can you determine about the degree of the difference of Janie's and Max's polynomials?

3. Suppose Janie writes a polynomial expression using only one variable, x, with degree of 5, and Max writes a polynomial expression using only one variable, x, with degree of 5.
   a. What can you determine about the degree of the sum of Janie's and Max's polynomials?
   b. What can you determine about the degree of the difference of Janie's and Max's polynomials?

4. Find each sum or difference by combining the parts that are alike.
   a. $(2p+4) + 5(p-1) - (p+7)$
   b. $(7x^4 + 9x) - 2(x^4 + 13)$
   c. $(6 - t - t^4) + (9t + t^4)$
   d. $(5 - t^2) + 6(t^2 - 8) - (t^2 + 12)$
   e. $(8x^3 + 5x) - 3(x^3 + 2)$
   f. $(12x+1) + 2(x-4) - (x-15)$
   g. $(13x^2 + 5x) - 2(x^2 + 1)$
   h. $(9 - t - t^2) - \frac{3}{2}(8t + 2t^2)$
   i. $(4m+6) - 12(m-3) + (m+2)$
   j. $(15x^4 + 10x) - 12(x^4 + 4x)$