Lesson 10: True and False Equations

Classwork

Exercise 1

a. Consider the statement: "The president of the United States is a United States citizen."
Is the statement a grammatically correct sentence?
What is the subject of the sentence? What is the verb in the sentence?
Is the sentence true?

b. Consider the statement: "The president of France is a United States citizen."
Is the statement a grammatically correct sentence?
What is the subject of the sentence? What is the verb in the sentence?
Is the sentence true?

c. Consider the statement: "$2 + 3 = 1 + 4$."
This is a sentence. What is the verb of the sentence? What is the subject of the sentence?
Is the sentence true?

d. Consider the statement: "$2 + 3 = 9 + 4$."
Is this statement a sentence? And if so, is the sentence true or false?

A *number sentence* is a statement of equality between two numerical expressions.

A *number sentence* is said to be *true* if both numerical expressions are equivalent (that is, both evaluate to the same number). It is said to be *false* otherwise. True and false are called *truth values*.

Exercise 2

Determine whether the following number sentences are true or false.

a. $4 + 8 = 10 + 5$

b. $\frac{1}{2} + \frac{5}{8} = 1.2 - 0.075$

c. $(71 \cdot 603) \cdot 5876 = 603 \cdot (5876 \cdot 71)$

d. $13 \times 175 = 13 \times 90 + 85 \times 13$

e. $(7 + 9)^2 = 7^2 + 9^2$

f. $\pi = 3.141$

g. $\sqrt{(4 + 9)} = \sqrt{4} + \sqrt{9}$

h. $\frac{1}{2}+\frac{1}{3}=\frac{2}{5}$

i. $\frac{1}{2}+\frac{1}{3}=\frac{2}{6}$

j. $\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$

k. $3^2+4^2=7^2$

l. $3^2\times 4^2=12^2$

m. $3^2\times 4^3=12^6$

n. $3^2\times 3^3=3^5$

Exercise 3

a. Could a number sentence be both true and false?

b. Could a number sentence be neither true nor false?

An *algebraic equation* is a statement of equality between two expressions.

Algebraic equations can be number sentences (when both expressions are numerical), but often they contain symbols whose values have not been determined.

Exercise 4

a. Which of the following are algebraic equations?

i. $3.1x - 11.2 = 2.5x + 2.3$

ii. $10\pi^4 + 3 = 99\pi^2$

iii. $\pi + \pi = 2\pi$

iv. $\frac{1}{2} + \frac{1}{2} = \frac{2}{4}$

v. $79\pi^3 + 70\pi^2 - 56\pi + 87 = \frac{60\pi + 29\,928}{\pi^2}$

b. Which of them are also number sentences?

c. For each number sentence, state whether the number sentence is true or false.

Exercises 5

When algebraic equations contain a symbol whose value has not yet been determined, we use analysis to determine whether:

a. The equation is true for all the possible values of the variable(s), or
b. The equation is true for a certain set of the possible value(s) of the variable(s), or
c. The equation is never true for any of the possible values of the variable(s).

For each of the three cases, write an algebraic equation that would be correctly described by that case. Use only the variable, x, where x represents a real number.

Example 1

Consider the following scenario.

Julie is 300 feet away from her friend's front porch and observes, "Someone is sitting on the porch."

Given that she did not specify otherwise, we would assume that the *someone* Julie thinks she sees is a human. We cannot guarantee that Julie's observational statement is true. It could be that Julie's friend has something on the porch that merely looks like a human from far away. Julie assumes she is correct and moves closer to see if she can figure out who it is. As she nears the porch, she declares, "Ah, it is our friend, John Berry."

Exercise 6

Name a value of the variable that would make each equation a true number sentence.

Here are several examples of how we can name the value of the variable:

Let $w = -2$. Then, $w^2 = 4$ is true.

$w^2 = 4$ is true when $w = -2$.

$w^2 = 4$ is true if $w = -2$.

$w^2 = 4$ is true for $w = -2$ and $w = 2$.

There might be more than one option for what numerical values to write. (And feel free to write more than one possibility.)

Warning: Some of these are tricky. Keep your wits about you!

a. Let _____. Then, $7 + x = 12$ is true.

b. Let _____. Then, $3r + 0.5 = \frac{37}{2}$ is true.

c. $m^3 = -125$ is true for _____.

d. A number x and its square, x^2, have the same value when _____.

e. The average of 7 and n is -8 if _____.

f. Let _____. Then, $2a = a + a$ is true.

g. $q + 67 = q + 68$ is true for _____.

Problem Set

Determine whether the following number sentences are true or false.

1. $18 + 7 = \frac{50}{2}$
2. $3.123 = 9.369 \cdot \frac{1}{3}$
3. $(123 + 54) \cdot 4 = 123 + (54 \cdot 4)$
4. $5^2 + 12^2 = 13^2$
5. $(2 \times 2)^2 = \sqrt{256}$
6. $\frac{4}{3} = 1.333$

In the following equations, let $x = -3$ and $y = \frac{2}{3}$. Determine whether the following equations are true, false, or neither true nor false.

7. $xy = -2$
8. $x + 3y = -1$
9. $x + z = 4$
10. $9y = -2x$
11. $\frac{y}{x} = -2$
12. $\frac{-\frac{2}{x}}{y} = -1$

For each of the following, assign a value to the variable, x, to make the equation a true statement.

13. $(x^2 + 5)(3 + x^4)(100x^2 - 10)(100x^2 + 10) = 0$ for _____.
14. $\sqrt{(x + 1)(x + 2)} = \sqrt{20}$ for _____.
15. $(d + 5)^2 = 36$ for _____.
16. $(2z + 2)(z^5 - 3) + 6 = 0$ for _____.
17. $\frac{1+x}{1+x^2} = \frac{3}{5}$ for _____.
18. $\frac{1+x}{1+x^2} = \frac{2}{5}$ for _____.
19. The diagonal of a square of side length L is 2 inches long when _____.
20. $\left(T - \sqrt{3}\right)^2 = T^2 + 3$ for _____.
21. $\frac{1}{x} = \frac{x}{1}$ if _____.
22. $\left(2 + \left(2 - \left(2 + \left(2 - (2 + r)\right)\right)\right)\right) = 1$ for _____.

23. $x + 2 = 9$

24. $x + 2^2 = -9$

25. $-12t = 12$

26. $12t = 24$

27. $\frac{1}{b-2} = \frac{1}{4}$

28. $\frac{1}{2b-2} = -\frac{1}{4}$

29. $\sqrt{x} + \sqrt{5} = \sqrt{x+5}$

30. $(x-3)^2 = x^2 + (-3)^2$

31. $x^2 = -49$

32. $\frac{2}{3} + \frac{1}{5} = \frac{3}{x}$

Fill in the blank with a variable term so that the given value of the variable will make the equation true.

33. $\_\_\_\_\_ + 4 = 12; x = 8$

34. $\_\_\_\_\_ + 4 = 12; x = 4$

Fill in the blank with a constant term so that the given value of the variable will make the equation true.

35. $4y - \_\_\_\_ = 100; y = 25$

36. $4y - \_\_\_\_ = 0; y = 6$

37. $r + \_\_\_\_ = r$; r is any real number.

38. $r \times \_\_\_\_ = r$; r is any real number.

Generate the following:

39. An equation that is always true

40. An equation that is true when $x = 0$

41. An equation that is never true

42. An equation that is true when $t = 1$ or $t = -1$

43. An equation that is true when $y = -0.5$

44. An equation that is true when $z = \pi$