Lesson 14: Solving Inequalities

Classwork

Exercise 1

1. Consider the inequality $x^2 + 4x \geq 5$.

   a. Sift through some possible values to assign to x that make this inequality a true statement. Find at least two positive values that work and at least two negative values that work.

   b. Should your four values also be solutions to the inequality $x(x+4) \geq 5$? Explain why or why not. Are they?

   c. Should your four values also be solutions to the inequality $4x + x^2 \geq 5$? Explain why or why not. Are they?

   d. Should your four values also be solutions to the inequality $4x + x^2 - 6 \geq -1$? Explain why or why not. Are they?

   e. Should your four values also be solutions to the inequality $12x + 3x^2 \geq 15$? Explain why or why not. Are they?

Example 1

What is the solution set to the inequality $5q + 10 > 20$? Express the solution set in words, in set notation, and graphically on the number line.

Exercises 2–3

2. Find the solution set to each inequality. Express the solution in set notation and graphically on the number line.

   a. $x + 4 \leq 7$

   b. $\frac{m}{3} + 8 \neq 9$

   c. $8y + 4 < 7y - 2$

   d. $6(x - 5) \geq 30$

   e. $4(x - 3) > 2(x - 2)$

3. Recall the discussion on all the strange ideas for what could be done to both sides of an equation. Let's explore some of the same issues here but with inequalities. Recall, in this lesson, we have established that adding (or subtracting) and multiplying through by positive quantities does not change the solution set of an inequality. We've made no comment about other operations.

   a. Squaring: Do $B \leq 6$ and $B^2 \leq 36$ have the same solution set? If not, give an example of a number that is in one solution set but not the other.

   b. Multiplying through by a negative number: Do $5 - C > 2$ and $-5 + C > -2$ have the same solution set? If not, give an example of a number that is in one solution set but not the other.

   c. Bonzo's ignoring exponents: Do $y^2 < 5^2$ and $y < 5$ have the same solution set?

Example 2

Jojo was asked to solve $6x + 12 < 3x + 6$, for x. She answered as follows:

$6x + 12 < 3x + 6$

$6(x + 2) < 3(x + 2)$ Apply the distributive property.

$6 < 3$ Multiply through by $\frac{1}{x+2}$.

a. Since the final line is a false statement, she deduced that there is no solution to this inequality (that the solution set is empty).

   What is the solution set to $6x + 12 < 3x + 6$?

b. Explain why Jojo came to an erroneous conclusion.

Example 3

Solve $-q \geq -7$, for q.

Exercises 4–7

4. Find the solution set to each inequality. Express the solution in set notation and graphically on the number line.

   a. $-2f < -16$

   b. $-\frac{x}{12} \leq \frac{1}{4}$

   c. $6 - a \geq 15$

   d. $-3(2x + 4) > 0$

Recall the properties of inequality:

- Addition property of inequality:
  If $A > B$, then $A + c > B + c$ for any real number c.

- Multiplication property of inequality:
  If $A > B$, then $kA > kB$ for any <u>positive</u> real number k.

5. Use the properties of inequality to show that each of the following is true for any real numbers p and q.

   a. If $p \geq q$, then $-p \leq -q$.

   b. If $p < q$, then $-5p > -5q$.

c. If $p \leq q$, then $-0.03p \geq -0.03q$.

d. Based on the results from parts (a) through (c), how might we expand the multiplication property of inequality?

6. Solve $-4 + 2t - 14 - 18t > -6 - 100t$, for t in two different ways: first without ever multiplying through by a negative number and then by first multiplying through by $-\frac{1}{2}$.

7. Solve $-\frac{x}{4} + 8 < \frac{1}{2}$, for x in two different ways: first without ever multiplying through by a negative number and then by first multiplying through by -4.

Problem Set

1. Find the solution set to each inequality. Express the solution in set notation and graphically on the number line.
   a. $2x < 10$
   b. $-15x \geq -45$
   c. $\frac{2}{3}x \neq \frac{1}{2} + 2$
   d. $-5(x - 1) \geq 10$
   e. $13x < 9(1 - x)$

2. Find the mistake in the following set of steps in a student's attempt to solve $5x + 2 \geq x + \frac{2}{5}$, for x. What is the correct solution set?

   $5x + 2 \geq x + \frac{2}{5}$

   $5\left(x + \frac{2}{5}\right) \geq x + \frac{2}{5}$ (factoring out 5 on the left side)

   $5 \geq 1$ (dividing by $\left(x + \frac{2}{5}\right)$)

   So, the solution set is the empty set.

3. Solve $-\frac{x}{16} + 1 \geq -\frac{5x}{2}$, for x without multiplying by a negative number. Then, solve by multiplying through by -16.

4. Lisa brought half of her savings to the bakery and bought 12 croissants for $\$14.20$. The amount of money she brings home with her is more than $\$2.00$. Use an inequality to find how much money she had in her savings before going to the bakery. (Write the inequality that represents the situation, and solve it.)