Lesson 21: Solution Sets to Inequalities with Two Variables

Classwork

Exercises 1–2

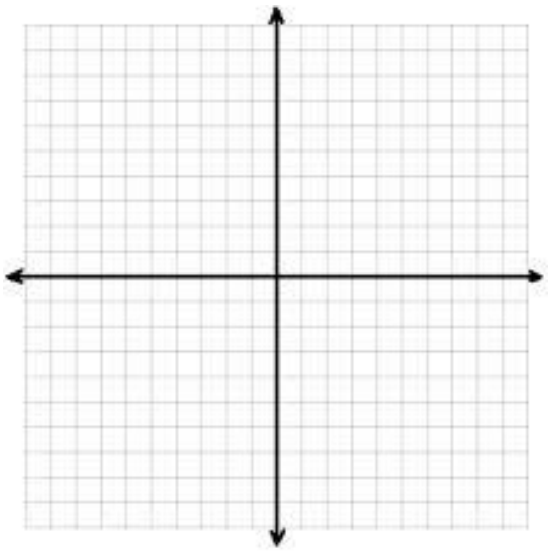
1.

a. Circle each ordered pair (x, y) that is a solution to the equation $4x - y \leq 10$.

i. $(3,2)$ $(2, 3)$ $(-1, -14)$ $(0,0)$ $(1, -6)$

ii. $(5,10)$ $(0, -10)$ $(3,4)$ $(6,0)$ $(4, -1)$

b. Plot each solution as a point (x, y) in the coordinate plane.

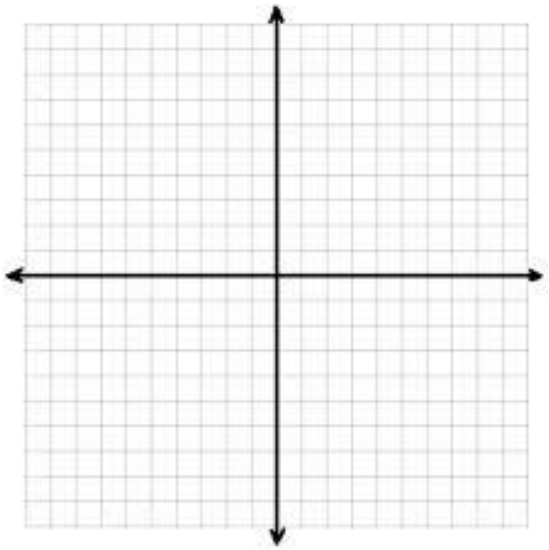


c. How would you describe the location of the solutions in the coordinate plane?

2.

a. Discover as many additional solutions to the inequality $4x - y \leq 10$ as possible. Organize solutions by plotting each solution as a point (x, y) in the coordinate plane. Be prepared to share the strategies used to find the solutions.

b. Graph the line $y = 4x - 10$. What do we notice about the solutions to the inequality $4x - y \leq 10$ and the graph of the line $y = 4x - 10$?



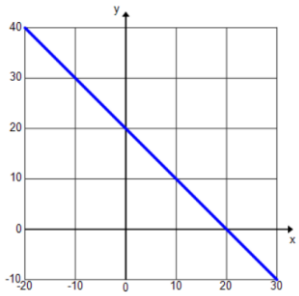
c. Solve the inequality for y.

d. Complete the following sentence:
If an ordered pair is a solution to $4x - y \leq 10$, then it will be located _____ the line $y = 4x - 10$.

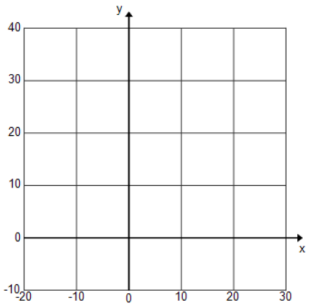
e. Explain how you arrived at your conclusion.

Example

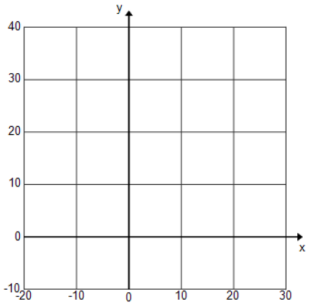
The solution to $x + y = 20$ is shown on the graph below.



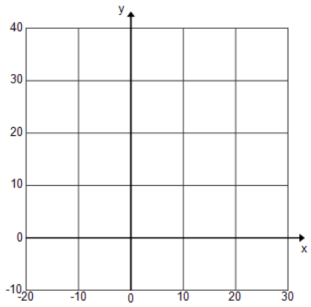
a. Graph the solution to $x + y \leq 20$.



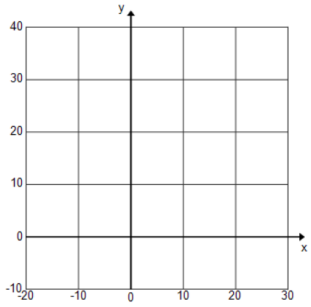
b. Graph the solution to $x + y \geq 20$.



c. Graph the solution to $x + y < 20$.



d. Graph the solution to $x + y > 20$.



Exercises 3–5

3. Using a separate sheet of graph paper, plot the solution sets to the following equations and inequalities:

a. $x - y = 10$	f. $y = 5$	k. $x > 0$
b. $x - y < 10$	g. $y < 5$	l. $y < 0$
c. $y > x - 10$	h. $x \geq 5$	m. $x^2 - y = 0$
d. $y \geq x$	i. $y \neq 1$	n. $x^2 + y^2 > 0$
e. $x \geq y$	j. $x = 0$	o. $xy \leq 0$

Which of the inequalities in this exercise are *linear* inequalities?

A *half-plane* is the graph of a solution set in the Cartesian coordinate plane of an inequality in two real-number variables that is linear and strict.

4. Describe in words the half-plane that is the solution to each inequality.

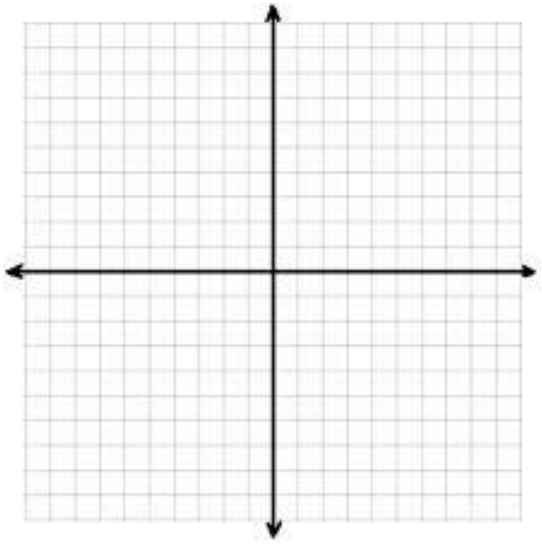
a. $y \geq 0$

b. $x < -5$

c. $y \geq 2x - 5$

d. $y < 2x - 5$

5. Graph the solution set to $x < -5$, reading it as an inequality in *one* variable, and describe the solution set in words. Then graph the solution set to $x < -5$ again, this time reading it as an inequality in *two* variables, and describe the solution set in words.



Lesson Summary

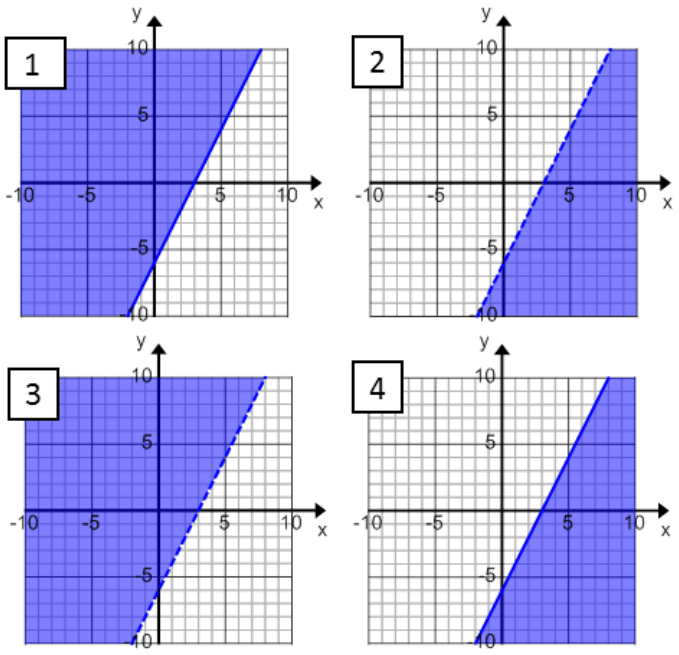
An ordered pair is a *solution* to a two-variable inequality if, when each number is substituted into its corresponding variable, it makes the inequality a true number sentence.

Each ordered pair of numbers in the solution set of the inequality corresponds to a point on the coordinate plane. The set of all such points in the coordinate plane is called the *graph of the inequality*.

The graph of a linear inequality in the coordinate plane is called a *half-plane.*

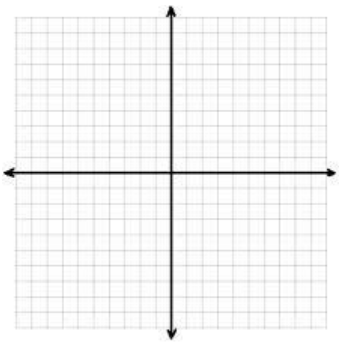
Problem Set

1. Match each inequality with its graph. Explain your reasoning.
   a. $2x - y > 6$
   b. $y \leq 2x - 6$
   c. $2x < y + 6$
   d. $2x - 6 \leq y$

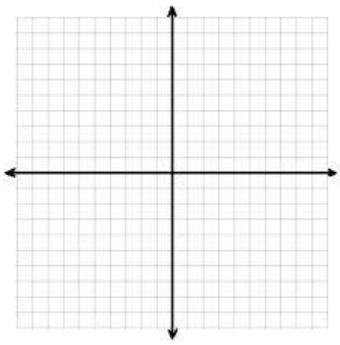


2. Graph the solution set in the coordinate plane. Support your answer by selecting two ordered pairs in the solution set and verifying that they make the inequality true.

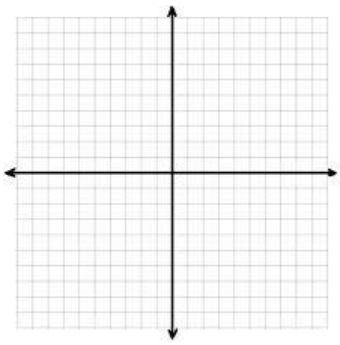
a. $-10x + y > 25$



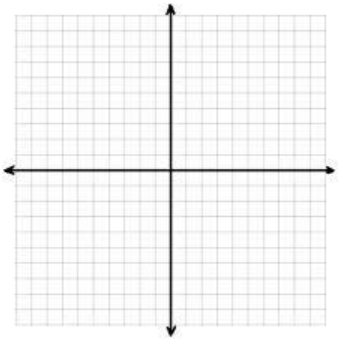
b. $-6 \leq y$



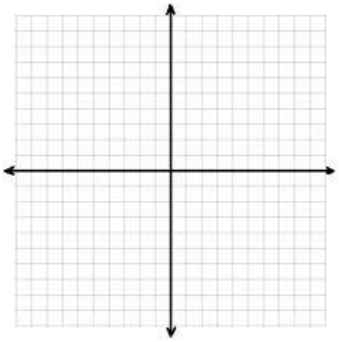
c. $y \leq -7.5x + 15$



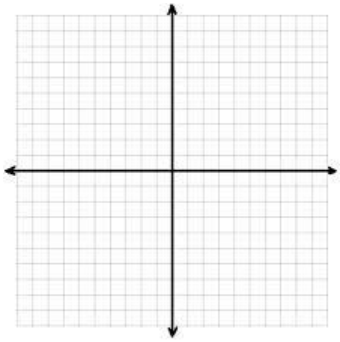
d. $2x - 8y \leq 24$



e. $3x < y$



f. $2x > 0$



3. Marti sells tacos and burritos from a food truck at the farmers market. She sells burritos for $\$3.50$ each and tacos for $\$2.00$ each. She hopes to earn at least $\$120$ at the farmers market this Saturday.

   a. Identify three combinations of tacos and burritos that will earn Marti more than $\$120$.

   b. Identify three combinations of tacos and burritos that will earn Marti exactly $\$120$.

   c. Identify three combinations of tacos and burritos that will *not* earn Marti at least $\$120$.

   d. Graph your answers to parts (a)–(c) in the coordinate plane, and then shade a half-plane that contains all possible solutions to this problem.

   e. Create a linear inequality that represents the solution to this problem. Let x equal the number of burritos that Marti sells, and let y equal the number of tacos that Marti sells.

   f. Is the point $(10, 49.5)$ a solution to the inequality you created in part (e)? Explain your reasoning.