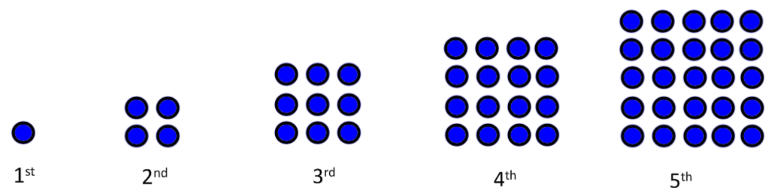
Lesson 8: Why Stay with Whole Numbers?

Classwork

Opening Exercise

The sequence of perfect squares $\{1,4,9,16,25,\ldots\}$ earned its name because the ancient Greeks realized these quantities could be arranged to form square shapes.



If $S(n)$ denotes the n^{th} square number, what is a formula for $S(n)$?

Exercises

1. Prove whether or not 169 is a perfect square.

2. Prove whether or not 200 is a perfect square.

3. If $S(n) = 225$, then what is n?

4. Which term is the number 400 in the sequence of perfect squares? How do you know?

Instead of arranging dots into squares, suppose we extend our thinking to consider squares of side length x cm.

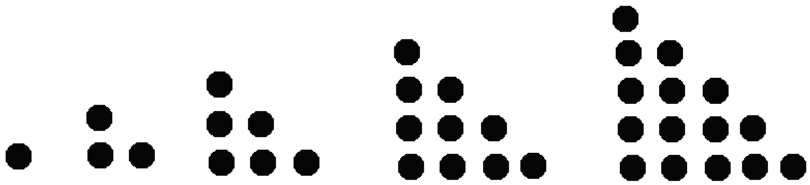
5. Create a formula for the area $A(x)\ \mathrm{cm}^2$ of a square of side length x cm: $A(x) =$ ___________.

6. Use the formula to determine the area of squares with side lengths of 3 cm, 10.5 cm, and π cm.

7. What does $A(0)$ mean?

8. What does $A(-10)$ and $A(\sqrt{2})$ mean?

The triangular numbers are the numbers that arise from arranging dots into triangular figures as shown:



9. What is the 100^{th} triangular number?

10. Find a formula for $T(n)$, the n^{th} triangular number (starting with $n = 1$).

11. How can you be sure your formula works?

12. Create a graph of the sequence of triangular numbers $(n) = \frac{n(n+1)}{2}$, where n is a positive integer.

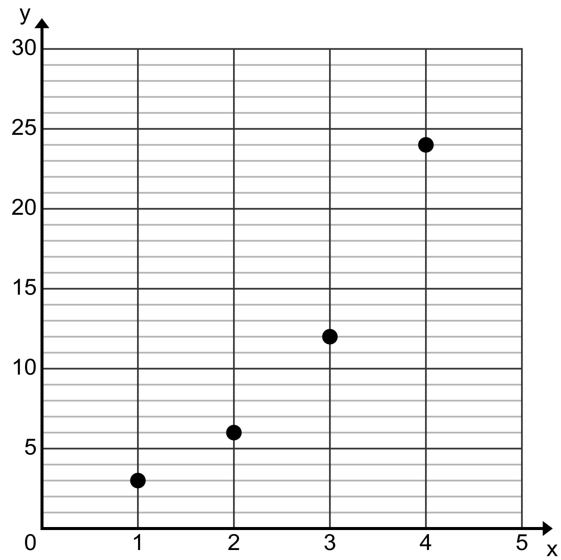
13. Create a graph of the triangle area formula $T(x) = \frac{x(x+1)}{2}$, where x is any positive real number.

14. How are your two graphs alike? How are they different?

Problem Set

1. The first four terms of two different sequences are shown below. Sequence A is given in the table, and sequence B is graphed as a set of ordered pairs.

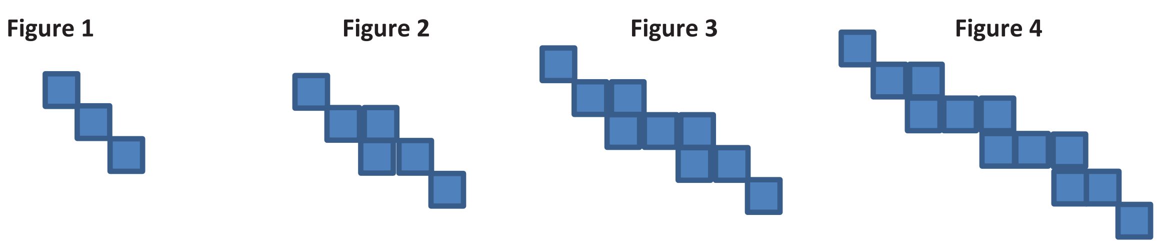
n	$A(n)$
1	15
2	31
3	47
4	63



   a. Create an explicit formula for each sequence.

   b. Which sequence will be the first to exceed 500? How do you know?

2. A tile pattern is shown below.

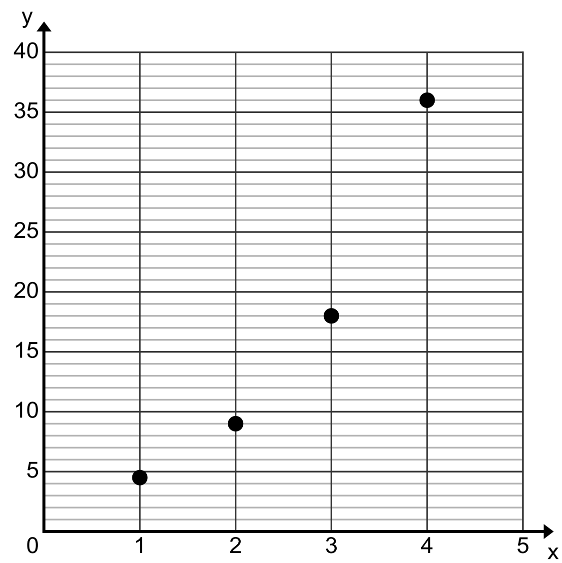


   a. How is this pattern growing?

   b. Create an explicit formula that could be used to determine the number of squares in the n^{th} figure.

   c. Evaluate your formula for $n = 0$, and $n = 2.5$. Draw Figure 0 and Figure 2.5, and explain how you decided to create your drawings.

3. The first four terms of a geometric sequence are graphed as a set of ordered pairs.



a. What is an explicit formula for this sequence?

b. Explain the meaning of the ordered pair $(3, 18)$.

c. As of July 2013, Justin Bieber had over $42{,}000{,}000$ Twitter followers. Suppose the sequence represents the number of people that follow your new Twitter account each week since you started tweeting. If your followers keep growing in the same manner, when will you exceed $1{,}000{,}000$ followers?