

Lesson 18: Four Interesting Transformations of Functions

Classwork

Example

Let $f(x) = |x|$, $g(x) = f(x - 3)$, and $h(x) = f(x + 2)$, where x can be any real number.

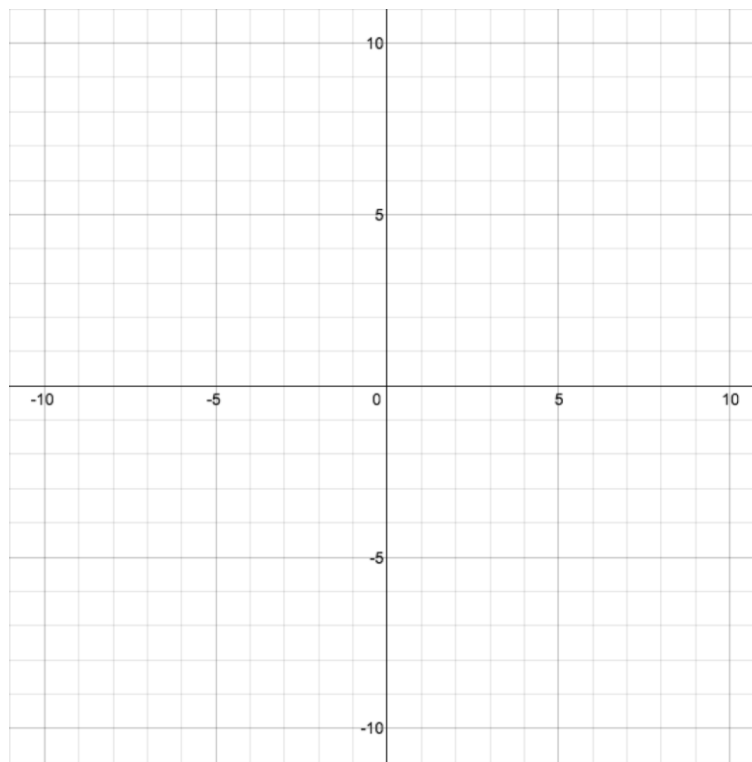
- Write the formula for $g(x)$ in terms of $|x|$ (i.e., without using $f(x)$ notation).

- Write the formula for $h(x)$ in terms of $|x|$ (i.e., without using $f(x)$ notation).

- Complete the table of values for these functions.

x	$f(x) = x $	$g(x) =$	$h(x) =$
-3			
-2			
-1			
0			
1			
2			
3			

- d. Graph all three equations: $y = f(x)$, $y = f(x - 3)$, and $y = f(x + 2)$.



- e. How does the graph of $y = f(x)$ relate to the graph of $y = f(x - 3)$?
- f. How does the graph of $y = f(x)$ relate to the graph of $y = f(x + 2)$?
- g. How do the graphs of $y = |x| - 3$ and $y = |x - 3|$ relate differently to the graph of $y = |x|$?
- h. How do the values of g and h relate to the values of f ?

Exercises

1. Karla and Isamar are disagreeing over which way the graph of the function $g(x) = |x + 3|$ is translated relative to the graph of $f(x) = |x|$. Karla believes the graph of g is “to the right” of the graph of f ; Isamar believes the graph is “to the left.” Who is correct? Use the coordinates of the vertex of f and g to support your explanation.

2. Let $f(x) = |x|$, where x can be any real number. Write a formula for the function whose graph is the transformation of the graph of f given by the instructions below.
 - a. A translation right 5 units

 - b. A translation down 3 units

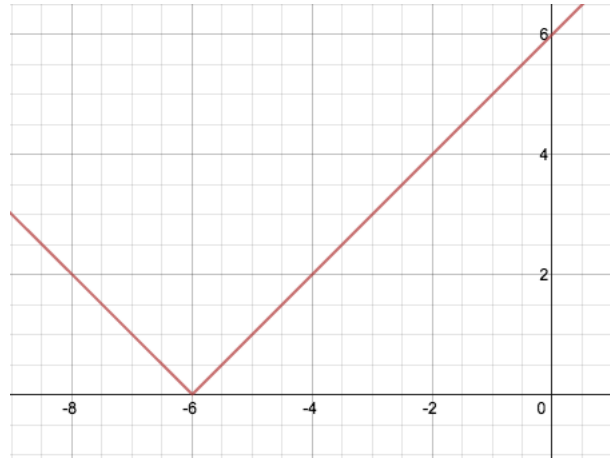
 - c. A vertical scaling (a vertical stretch) with scale factor of 5

 - d. A translation left 4 units

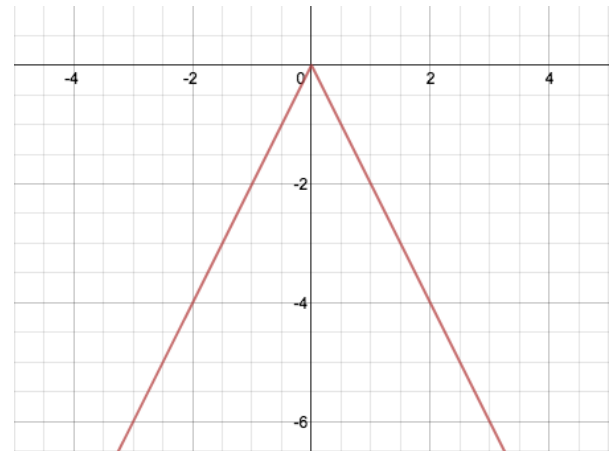
 - e. A vertical scaling (a vertical shrink) with scale factor of $\frac{1}{3}$

3. Write the formula for the function depicted by the graph.

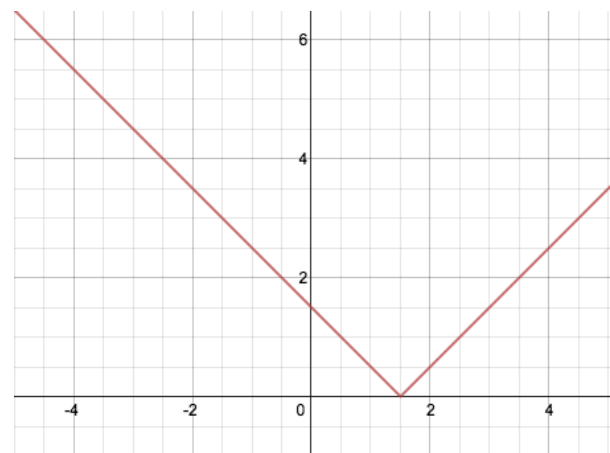
a. $y =$



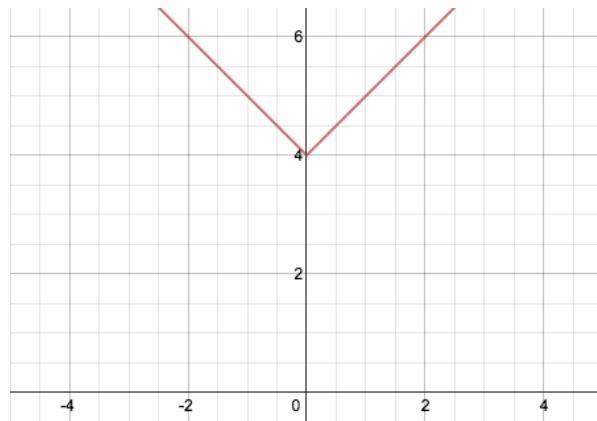
b. $y =$



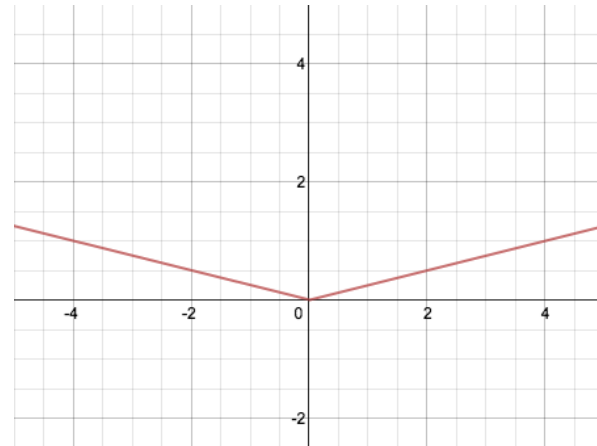
c. $y =$



d. $y =$



e. $y =$



4. Let $f(x) = |x|$, where x can be any real number. Write a formula for the function whose graph is the described transformation of the graph of f .

a. A translation 2 units left and 4 units down

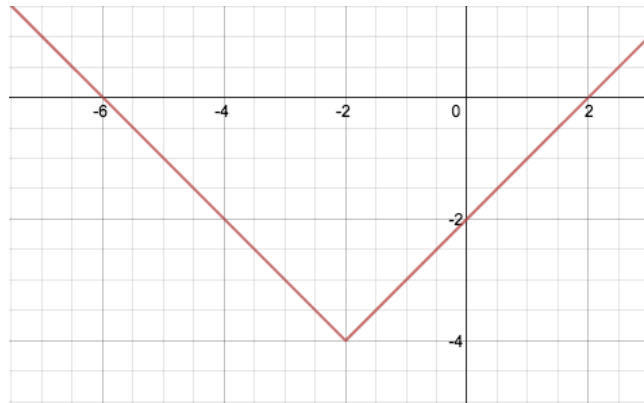
b. A translation 2.5 units right and 1 unit up

c. A vertical scaling with scale factor $\frac{1}{2}$ and then a translation 3 units right

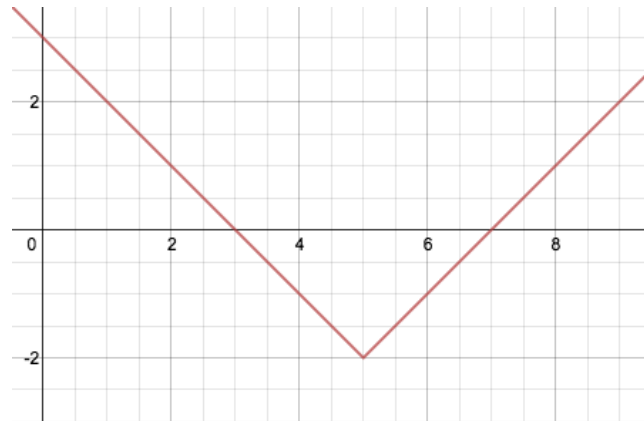
d. A translation 5 units right and a vertical scaling by reflecting across the x -axis with vertical scale factor -2

5. Write the formula for the function depicted by the graph.

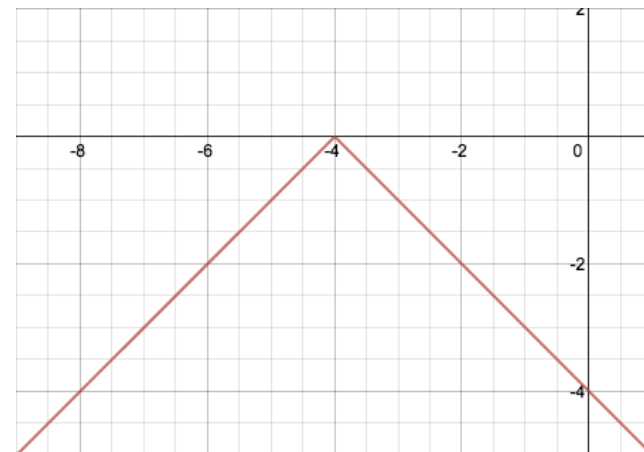
a. $y =$



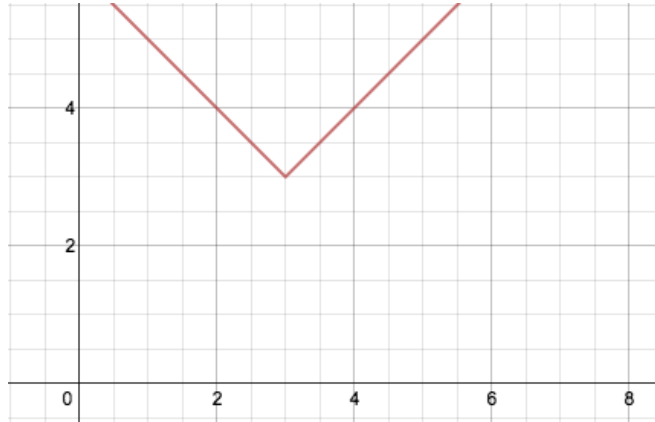
b. $y =$



c. $y =$



d. $y =$



Problem Set

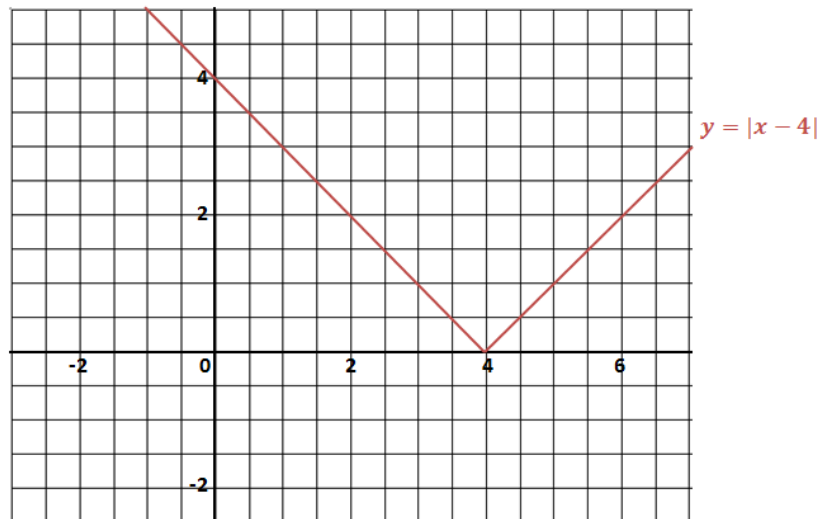
1. Working with quadratic functions:

- a. The vertex of the quadratic function $f(x) = x^2$ is at $(0,0)$, which is the minimum for the graph of f . Based on your work in this lesson, to where do you predict the vertex will be translated for the graphs of $g(x) = (x - 2)^2$ and $h(x) = (x + 3)^2$?
- b. Complete the table of values, and then graph all three functions.

x	$f(x) = x^2$	$g(x) = (x - 2)^2$	$h(x) = (x + 3)^2$
-3			
-2			
-1			
0			
1			
2			
3			

2. Let $f(x) = |x - 4|$ for every real number x . The graph of the equation $y = f(x)$ is provided on the Cartesian plane below. Transformations of the graph of $y = f(x)$ are described below. After each description, write the equation for the transformed graph. Then, sketch the graph of the equation you write for part (d).

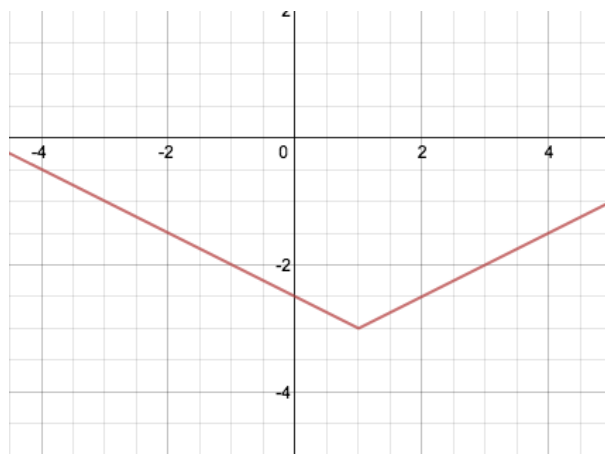
- a. Translate the graph left 6 units and down 2 units.
- b. Reflect the resulting graph from part (a) across the x -axis.
- c. Scale the resulting graph from part (b) vertically by a scale factor of $\frac{1}{2}$.
- d. Translate the resulting graph from part (c) right 3 units and up 2 units. Graph the resulting equation.



3. Let $f(x) = |x|$ for all real numbers x . Write the formula for the function represented by the described transformation of the graph of $y = f(x)$.
- First, a vertical stretch with scale factor $\frac{1}{3}$ is performed, then a translation right 3 units, and finally a translation down 1 unit.
 - First, a vertical stretch with scale factor 3 is performed, then a reflection over the x -axis, then a translation left 4 units, and finally a translation up 5 units.
 - First, a reflection across the x -axis is performed, then a translation left 4 units, then a translation up 5 units, and finally a vertical stretch with scale factor 3.
 - Compare your answers to parts (b) and (c). Why are they different?

4. Write the formula for the function depicted by each graph.

a. $a(x) =$



b. $b(x) =$

