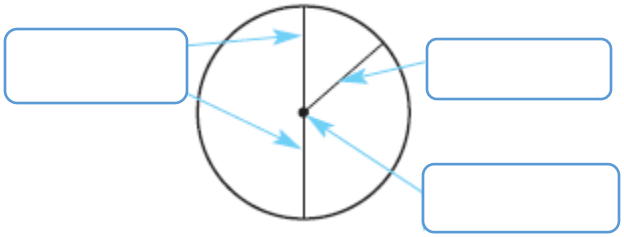
Notes 8.1 Circumference

A **circle** is the set of all points in a plane that are the same distance from a point called the **center**. The **circumference** is the distance around a circle. The **diameter** is the distance across a circle through its center. The **radius** is the distance from the center to any point on the circle.

Fill in each box with one of the following terms: *center, diameter,* and *radius.*



Radius and Diameter

Words The diameter, *d*, of a circle is twice its radius, *r*. The radius, *r*, of a circle is half of its diameter, *d*.

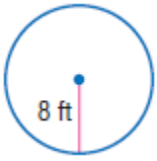
Symbols $d = 2r$ $r = \frac{d}{2}$

Example 1: The diameter of a circle is 14 inches. Find the radius.

$r = \frac{d}{2}$	Radius of circle formula.
$r = \frac{14}{2}$	Substitute *d* with 14.
$r = 7$	Divide.

The radius is 7 inches.

Example 2: The radius of a circle is 8 feet. Find the diameter.



$d = 2r$	Diameter of circle formula.
$d = 2 \bullet 8$	Substitute *r* with 8.
$d = 16$	Multiply.

The diameter is 16 feet.

Try These: Find the radius or diameter of each circle with the given dimensions.

a. $d = 23\ cm$
$r = ?$

b. $r = 3\ in.$
$d = ?$

c. $d = 16\ yd$
$r = ?$

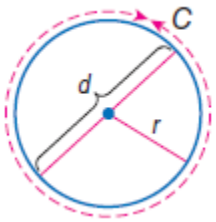
d. $r = 5.2\ m$
$d = ?$

Circumference

Words The circumference of a circle is equal to π times its diameter or π times twice its radius.

Symbols $C = \pi d \ \ or \ \ C = 2\pi r$

Model



*** Use the π button on the calculator when solving problems involving π.**

*** When rounding, use the "$\approx$" (approximately equal to) symbol.**

*** In Terms of pi (π): When asked to leave an answer is terms of pi, it simply means to give the answer (with the label) before multiplying by π.**

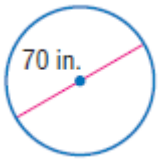
Example 3: Find the circumference of a circle with a radius of 21 inches. State your answer in terms of π and to the nearest hundredth.

$C = 2\pi r$	Circumference of a circle.
$C = 2 \bullet \pi \bullet 21$	Substitute *r* with 21.
$C = 42 \bullet \pi$	Multiply 2 and 21.
$C = 42\ \pi\ in$	Answer in terms of π.
$C \approx 131.946$	Multiply π button by 42.
$C \approx 131.95$	Round to nearest hundredth.

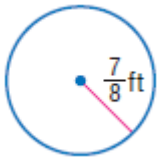
The circumference of the circle is about 131.95 inches.

Try These: Find the circumference of each circle. State your answer in terms of π and to the nearest hundredth.

e.



f.



Example 4: Big Ben is a famous clock tower in London, England. The diameter of the clock face is 23 feet. Find the circumference of the clock face. State your answer in terms of π and to the nearest hundredth.

$C = \pi d$	Circumference of a circle.
$C = \pi \bullet 23$	Substitute *d* with 23.
$C = 23\pi \ feet$	Answer is terms of π.
$C \approx 72.256$	Multiply.
$C \approx 72.26$	Round to the nearest hundredth.

The circumference of the clock face is about 72.26 feet.

Try This:

g. A circular fence is being placed to surround a tree. The diameter of the fenced area is 4 feet. How much fencing is used? Round to the nearest hundredth.

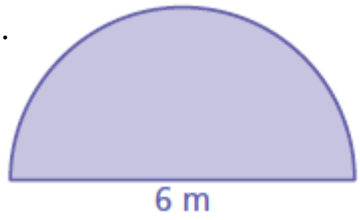
Notes 8.2 Circumference and Semicircles

Finding the Perimeter of a Semicircular Region

A **semicircle** is one-half of a circle. Use the formula for the circumference of a circle and divide it by 2.

Example 1: Find the perimeter of the semicircular region.

The straight side is 6 meters long. The distance around the curved part is one-half the circumference of a circle with a diameter of 6 meters. Round to the nearest hundredth.

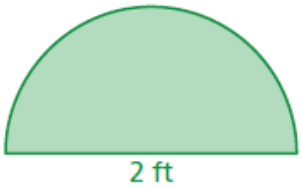


$C = \pi d$	Circumference formula.
$C = \frac{\pi d}{2}$	Divide circle formula by 2.
$P = \frac{\pi d}{2} + d$	Add the diameter.
$P = \frac{\pi \cdot 6}{2} + 6$	Substitute 6 in for d.
$P \approx 9.42 + 6$	Multiply π by 6 and divide by 2.
$P \approx 15.42$	Add. Round to the nearest hundredth.

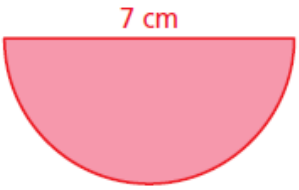
The perimeter is about 15.42 meters.

Try These: Find the perimeter of the semicircles. Round to the nearest hundredth.

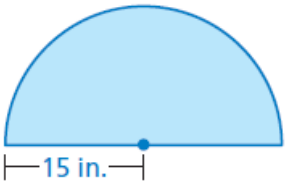
a.



b.



c.

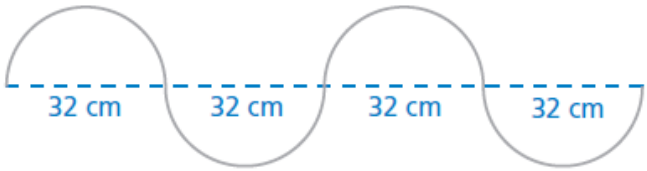


Class Practice:

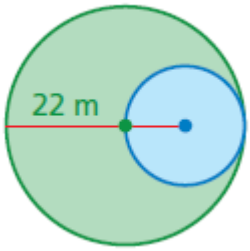
1. Find the circumference of the watch face. Round to the nearest tenth.

2. Find the circumference of the pizza. Round to the nearest tenth.

3. A wire is bent to form four semicircles. How long is the wire? Round to the nearest tenth.

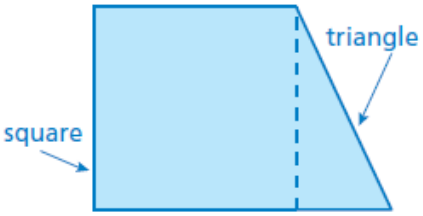


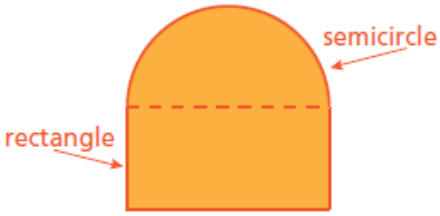
4. Find the circumference of both circles. Round to the nearest tenth.



Notes 8.3 Perimeter of Composite Figures

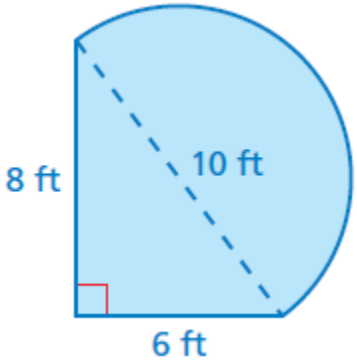
A **composite figure** is made up of triangles, squares, rectangles, semicircles, and other two-dimensional figures. Here are two examples.





To find the perimeter of a composite figure, find the distance around the **outside** of the entire figure.

Example 1: The figure is made up of a semicircle and a triangle. Find the perimeter.



The distance around the triangular part of the figure is 6 + 8 = 14 feet.

The distance around the semicircle is one-half the circumference of a circle with a diameter of 10 feet. Round to the nearest hundredth.

$C = \frac{\pi d}{2}$ Divide the circumference by 2.

$C = \frac{\pi 10}{2}$ Substitute 10 for *d*.

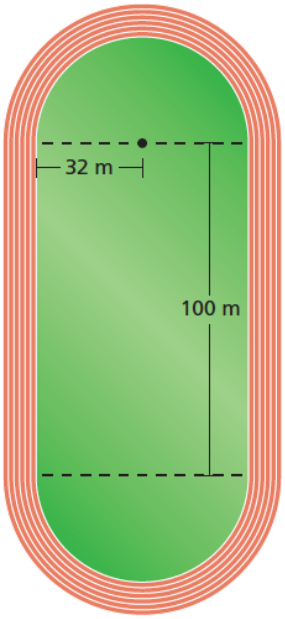
$C \approx 15.707$ Multiply π by 10 and then divide by 2.

$C \approx 15.71$ Round to the nearest hundredth.

So, the perimeter is about 14 + 15.71 ≈ 29.71 feet.

Example 2: The running track is made up of a rectangle and two semicircles. Find the perimeter. Round to the nearest hundredth.

The semicircular ends of the track form a circle with a radius of 32 meters. Find its circumference.

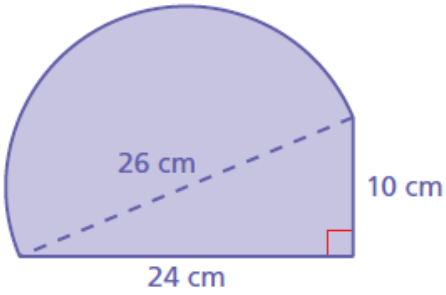


$C = 2\pi r$	Write formula for circumference.
$C \approx 2 \bullet \pi \bullet 32$	Substitute 32 for r.
$C \approx 201.061$	Multiply.
$C \approx 201.06$	Round to the nearest hundredth.

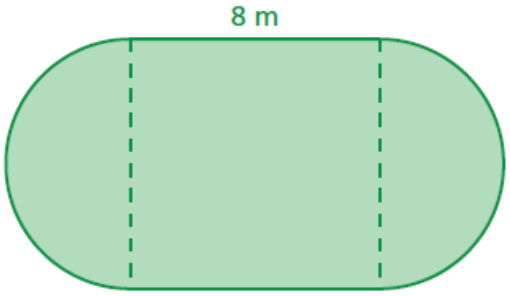
So, the perimeter is about 100 +100 + 201.06 ≈ 401.06 meters.

Try These: Find the perimeter of each figure. Round to the nearest hundredth.

c. The figure is made up of a semicircle and a triangle.



d. The figure is made up of a square and two semicircles.



Notes 8.4 Area of Circles

Find the Area of a Circle

Words The area, A, of a circle equals the product of π and the square of its radius, r.

Symbols $A = \pi r^2$

Model

Example 1: Find the area of the circle. State your answer in terms of π and to the nearest hundredth.

$A = \pi r^2$	Area of a circle.
$A = \pi \bullet 2^2$	Substitute 2 for r.
$A = \pi \bullet 4$	Multiply.
$A = 4\pi\ in^2$	Answer in terms of π.
$A \approx 12.566$	Multiply.
$A \approx 12.57$	Round to the nearest hundredth.

The area of the circle is approximately 12.57 square inches.

Try This:

a. Find the area of a circle with a radius of 3.2 centimeters. State your answer in terms of π and to the nearest hundredth.

Example 2. Find the area of the face of the Virginia quarter with a diameter of 24 millimeters. Round to the nearest hundredth.

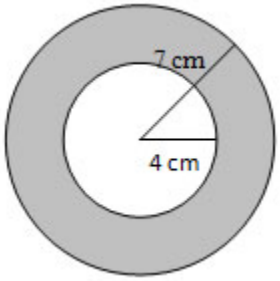
The radius is $\frac{1}{2}(24)$ or 12 millimeters.

$A = \pi r^2$	Area of a circle
$A = \pi \bullet 12^2$	Substitute 12 for *r*.
$A = \pi \bullet 144$	Multiply.
$A \approx 452.389$	Multiply.
$A \approx 452.39$	Round to the nearest hundredth.

The area is approximately 452.39 square millimeters.

Example 3: Find the area of the shaded region. Round to the nearest hundredth.

Step 1: Find the area of each circle.



Area of large circle:

$A = \pi r^2$

$A = \pi 7^2$

$A = \pi \bullet 49$

$A \approx 153.938$

$A \approx 153.94\ cm^2$

Area of small circle:

$A = \pi r^2$

$A = \pi 4^2$

$A = \pi \bullet 16$

$A \approx 50.265$

$A \approx 50.27 cm^2$

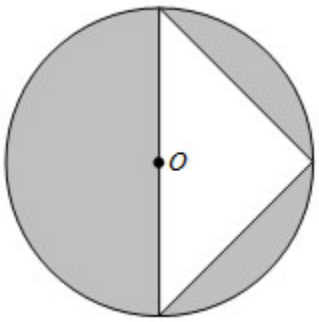
Step 2: Subtract the area of the smaller circle from the area of the larger circle.

$153.94 - 50.27 \approx 103.67$

The area of the shaded region is about 103.67 cm^2.

Try This:

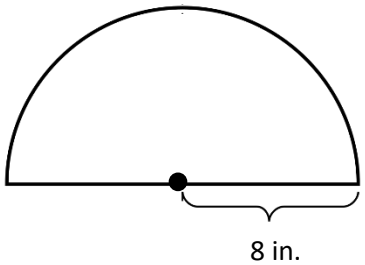
b. In the following figure, the triangle is an isosceles triangle with its base passing through the center of the circle. The diameter of the circle is 40 centimeters. Find the area of the shaded region. Round to the nearest hundredth.



Area of Semicircles

A **semicircle** is half of a circle. The formula for the area of a semicircle is $A = \frac{1}{2}\pi r^2$.

Example 4: Find the area of the semicircle. Round to the nearest hundredth.



$\boldsymbol{A = \frac{1}{2}\pi r^2}$	Area of a semicircle.
$A = \frac{1}{2}\pi \bullet 8^2$	Substitute 8 for *r*.
$A = \frac{1}{2}\pi \bullet 64$	Multiply.
$A = \pi \bullet 32$	Multiply.
$A \approx 100.530$	Multiply.
$A \approx 100.53$	Round to the nearest hundredth.

The area of the semicircle is approximately 100.53 square inches.

Try This:

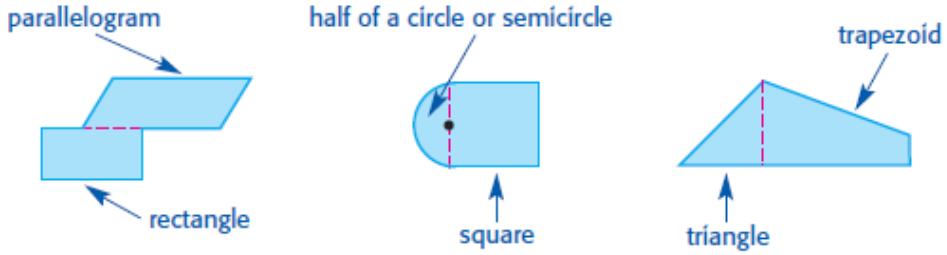
c. Find the approximate area of a semicircle with a diameter of 8 centimeters. Round to the nearest hundredth.

Notes 8.5 **Area of Composite Figures**

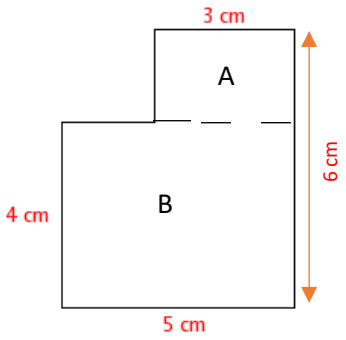
Find the Area of a Composite Figure

A **composite figure** is made up of two or more shapes. To find the area of a composite figure, decompose (separate) the figure into shapes with areas you know. Then find the sum of these areas.

Shape	Words	Formula
Parallelogram	The area, *A*, of a parallelogram is the product of any base, *b*, and its height, *h*.	$A = bh$
Triangle	The area, *A*, of a triangle is half the product of any base, *b*, and its height, *h*.	$A = \frac{1}{2}bh$
Trapezoid	The area, *A*, of a trapezoid is half the product of the height, *h*, and the sum of the bases,	$A = \frac{1}{2}h(b_1 + b_2)$
Circle	The area, *A*, of a circle is equal to π times the square of the radius, r.	$A = \pi r^2$



Example 1: Find the area of the composite figure.



The figure can be separated into two rectangles.

<u>Rectangle A</u>

$A = l \cdot w$

$A = 3 \cdot 2$

$A = 6 \text{ cm}^2$

<u>Rectangle B</u>

$A = l \cdot w$

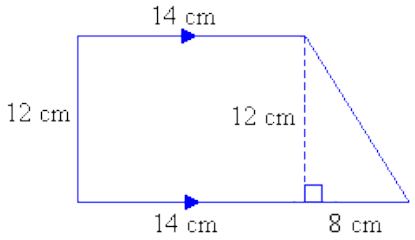
$A = 4 \cdot 5$

$A = 20 \text{ cm}^2$

Total Area = Area A + Area B

Total Area = 6 + 20 = 26 cm^2

Example 2: Find the area of the composite figure.



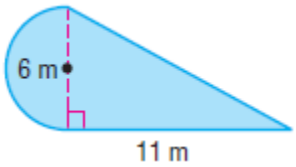
The figure can be separated into a rectangle and a triangle.

Rectangle

Triangle

Exampe 3: Find the area of the composite figure.

The figure can be separated into a semicircle and a triangle. Round to the nearest tenth.



Area of semicircle

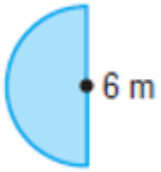
$A = \frac{1}{2}\pi r^2$

$A = \frac{1}{2}\pi \bullet 3^2$

$A \approx \frac{1}{2} \bullet \pi \bullet 9$

$A \approx 14.137$

$A \approx 14.1\ m^2$

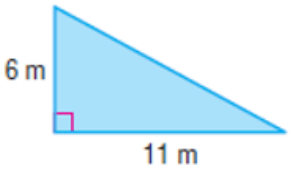


Area of triangle

$A = \frac{1}{2}bh$

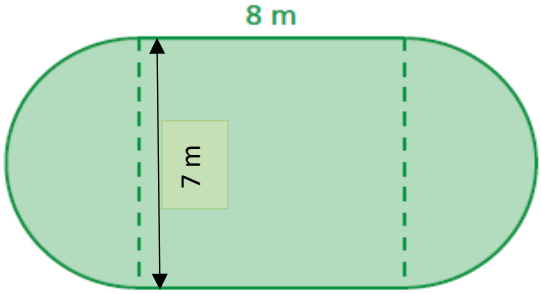
$A = \frac{1}{2} \bullet 11 \bullet 6$

$A = 33\ m^2$

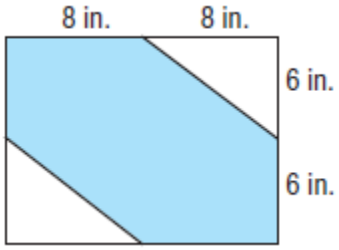


The area of the figure is about 14.1 + 33 or 47.1 square meters.

Try This: The running track is made up of a rectangle and two semicircles. Find the area of the track. Round to the nearest hundredth.



Example 4: **Two congruent triangles are cut from a rectangle. Find the area of the shaded region.**



Notes 8.6 Circumference to Area – Area to Circumference

<u>Circumference to Area:</u> Given the circumference of a circle, find the area of the circle.

Example 1: The circumference of a circular park is 450 feet. Find the area of the park. Round to the nearest hundredth.

Step 1: Find the radius of the park. Use the circumference formula.

$C = 2\pi r$	Circumference formula.
$450 = 2 \bullet \pi \bullet r$	Substitute 450 for C.
$\frac{450}{2} = \frac{2 \bullet \pi \bullet r}{2}$	Divide both sides by 2.
$\frac{225}{\pi} = \frac{\pi \bullet r}{\pi}$	Divide both sides by π.
$r \approx 71.619$	Divide.
$r \approx 71.62\ feet$	Round to the nearest hundredth.

Step 2: Find the area of the park. Use the area of a circle formula.

$A = \pi r^2$	Area of a circle formula.
$A = \pi \bullet (71.62)^2$	Substitute 71.62 in for r.
$A = \pi \bullet (5129.42)$	Multiply.
$A \approx 16{,}114.548$	Multiply.
$A \approx 16{,}114.55\ feet^2$	Multiply.

The area of the park is about 16,114.55 $feet^2$.

Try This:

a. **The circumference of a round dining table is 16.4 feet. Find the area of the dining table to the nearest hundredth of a foot.**

<u>Area to Circumference:</u> Given the area of a circle, find the circumference of the circle.

Example 2: The area of a round swimming pool is 1,230 square feet. Find the amount of fencing needed to enclose the pool. Round to the nearest hundredth.

Step 1: Find the radius of the pool. Use the area formula.

$A = \pi r^2$	Area of a circle formula.
$1230 = \pi \bullet r^2$	Substitute 1230 in for A.
$\frac{1230}{\pi} = \frac{\pi \bullet r^2}{\pi}$	Divide both sides by π.
$391.52116 \approx r^2$	Divide.
$\sqrt{391.52116} \approx \sqrt{r^2}$	Find the square root of each side.
$19.786 \approx r$	Simplify
$19.79\ ft \approx r$	Round to the nearest hundredth for the radius.

Step 2: Find the circumference of the pool. Use the circumference of a circle formula.

$C = 2\pi r$	Circumference of a circle formula.
$C = 2 \bullet \pi \bullet 19.79$	Substitute 19.80 in for r.
$C = 39.58 \bullet \pi$	Multiply.
$C \approx 124.344$	Multiply.
$C \approx 124.34\ ft$	Round to the nearest hundredth.

You need about 124.34 feet of fencing.

Try This:

b. **The town square is putting in a circular fountain. The fountain will have an area of 110 square feet. Find the distance around the edge of the fountain to the nearest hundredth of a foot.**