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Lesson 7.1: Factoring a GCF Algebra I

Factoring expressions is one of the **gateway skills** that is necessary for much of what we do in algebra for the rest of the course. The word **factor** has two meanings and both are important.

THE TWO MEANINGS OF FACTOR

1. **Factor (verb):** To rewrite an algebraic expression as an **equivalent product**.
2. **Factor (noun):** An algebraic expression that is one part of a larger factored expression.

Before we begin factoring, let's review the distributive property.

Exercise #1: Use the distributive property to rewrite $3x(2x+5)$.

Exercise #2: Factoring a GCF is essentially the opposite of distributing.

Let's look at factoring $6x^2+15x$

Step 1: What are some factors of $6x^2$? What are some factors of $15x$?

Step 2: What are some of their common factors?

Step 3: What is their greatest common factor (GCF)?

Step 4: Pull the GCF out of the expression by dividing all terms by the GCF.

Step 5: You can check you answer by distributing, you should get what you started with.

Exercise #3: Factor the following by pulling out a GCF.

a) $3x + 3y$

b) $8p - 8q$

c) $ab + ac$

d) $-3x - 15$

e) $14g + 7$

f) $4y + 6$

g) $z^2 - 4z$

h) $8a + 4b - 20c$

i) $w^3 - 3w^2 + w$

j) $5ab + 10a^2$

k) $-12xy^2 - 16x^2y$

l) $6x^3 - 8x^2 + 2x$

m) $8x^4 - 2x^2$

n) $-10x^2 - 40x - 50$

o) $8x^3 + 24x^2 - 32x$

Exercise #4: Rewrite each of the following expressions as the product of two binomials by factoring out a common binomial factor.

a) $(x+5)(x-1)+(x+5)(2x-3)$

b) $(2x-1)(2x+7)-(2x-1)(x-3)$

Exercise #5: Jordy factors the expression $16x-32$ and gets an expression in the form $a(b-c)$, what is the largest possible value of a?

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7.1: Factoring a GCF Algebra I

1. Identify the greatest common factor for each of the following sets of monomials.

a) $6x^2$ and $24x^3$ b) $2x^4$ and $10x^2$ c) $8t^5$, $12t^3$, and $16t$

2. Which of the following is the greatest common factor of the terms $36x^2y^4$ and $24xy^7$?

(1) $12xy^4$ (2) $24x^2y^7$

(3) $24x^2y^7$ (4) $3xy$

3. Write each of the following as equivalent products of the polynomials greatest common factor with another polynomial (pull out a GCF).

a) $50x + 30$ b) $x^2 - x$

c) $10x^2 + 35x - 20$ d) $4t^3 - 32t^2 + 12t$

4. Which of the following is *not* a correct factorization of the binomial $10x^2 + 40x$?

(1) $10x(x + 4)$ (2) $5x(2x + 4)$

(3) $10(x^2 + 4x)$ (4) $5x(2x + 8)$

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Lesson 7.2: Factoring Difference of Perfect Squares Algebra I

EX 1: a) Multiply $(x + 2)(x - 2)$

b) Multiply $(x + 5)(x - 5)$

c) So, if I give you $x^2 - 9$, what two factors did it come from?

d) And if I give you $x^2 - 64$, what two factors did it come from?

$x^2 - y^2$ is called a difference of two perfect squares. It always factors into $(x + y)(x - y)$. However, please be aware of the GCF and steps:

1. ____

2. ____

EX 2: Factor each binomial:

a) $a^2 - 49$

b) $32 - 2b^2$

c) $5 - 5c^2$

d) $d^2 - e^2$

e) $36 - 25f^2$

f) $g^2 - 144h^2$

g) $9j^2 - 100k^2$

h) $4m^2n^2 - 484$

i) $3p^4 - 147$

j) $196q^2 - r^4$

EX 3: Amelia believes that $x^6 - 81$ can be factored as $(x^4 - 9)(x^2 + 9)$. Her friend, Isabel believes that it can be factored as $(x^3 - 9)(x^3 + 9)$. Multiply out their respective factors to show which of the two friends has the correct factorization.

EX 4: A square is changed into a new rectangle by increasing its width by 2 inches and decreasing its length by 2 inches.

a) If the original square had a side length of 8 inches, find its area and the area of the new rectangle. Then, find how many square inches larger is the square's area?

b) If the original square had a side length of 20 inches, find its area and the area of the new rectangle. How many square inches larger is the square's area?

c) If the square had a side length of x inches, show that its area will always be four square inches more than the area of the new rectangle.

7.2 Hmwk

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Homework 7.2: Factoring Difference of Perfect Squares Algebra I

Factor the following expressions:

1. $2y^2 - 242$

2. $r^2 - 144$

3. $225 - m^2$

4. $9x^2 - 400$

5. $49 - 25q^2$

6. $9c^2 - 64d^2$

7. $48k^2 - 3$

8. $81u^2v^2 - 100w^2$

9. $25w^4 - 196$

10. $4 - 121e^4$

11. $169 - a^2b^4$

12. $g^6 - 36$

7.3 Notes

Name: ___________________________________ Date: __________________

Lesson 7.3: Factoring trinomials Algebra I

Factoring $x^2 + bx + c$

Review how to multiply:

a) $(x + 7)(x + 3)$

b) $(5 - a)(2 - a)$

c) $(2x + 3)(x + 1)$

d) $(3z + 2)(3z - 2)$

e) $(d + 7)^2$

f) $(2e - 5)^2$

Look at how this next problem is done:

$(x + 2)(x + 5)$

$x^2 \underline{+ 5x + 2x} + 10$ First Outside Inside Last

$x^2 + 7x + 10$ Combine like terms – the two middle terms

Looking at the numbers in the beginning of the problem (2 & 5), how do they relate to the number in the middle term of the answer?

Looking at the numbers in the beginning of the problem (2 & 5), how do they relate to the number in the last term of the answer?

THEREFORE, when you are working backward, you need to think of two numbers that

- ____________________________ to get that last term and **<u>the same two that</u>**
- ____________________________ to get the middle term.

KEEP IN MIND….the first piece you must factor is the ______________________________!!!

EX 1: Factor $x^2 + 3x + 2$.

EX 2: Factor:

a) $2x^2 + 16x + 30$

b) $20 + 9x + x^2$

c) $3x^2 - 18x + 15$

d) $7x^2 - 42x + 56$

e) $5x^2 + 10x - 15$

f) $x^2 + 3x - 4$

g) $x^4 - 5x^2 - 14$

h) $x^4 - 3x^2 - 18$

7.3 HW

Name: ________________________________ Date: ________________

HW 7.3: Factoring trinomials Algebra I

Factor each Trinomial

1. $3x^2 - 30x + 63$

2. $x^2 + 13x + 36$

3. $32 + 12x + x^2$

4. $4x^2 - 28x - 120$

5. $2x^2 - 10x - 48$

6. $x^2 + 3x + 2$

7. $x^2 + 3x - 18$

8. $8x^2 - 40x + 48$

9. $x^4 - 9x^2 + 20$

10. $x^4 + 7x^2 - 18$

11. $10x^2 - 80x - 90$

12. $x^2 + 7x + 10$

13. $x^2 + 21x + 38$

14. $y^2 - 18y + 45$

15. $11x^2 - 99x + 88$

16. $x^2 - 16x + 28$

7.4 Notes

Name: ________________________________ Date: ________________

Lesson 7.4: Factoring trinomials Algebra I

Factor:

a) $10c^2 + 60c + 90$

d) $2y^3 + 2y^2 - 220y$

b) $g^3 - 2g^2 - 24g$

e) $5x^3 - 25x^2 - 30x$

c) $x^2 - 11x + 18$

f) $x^2 + 13x - 48$

g) $3x^2 + 9x - 162$

i) $x^4 + 20x^2 + 100$

h) $12x^2 - 12x - 1080$

j) $x^6 - 15x^4 + 50x^2$

7.4 HW

Name: ________________________________ Date: ________________

HW 7.4: Factoring trinomials Algebra I

1. $2x^2 + 10x + 8$

2. $10x^2 + 200x + 1000$

3. $x^3 + 15x^2 + 50x$

4. $3a^3 - 15a^2 - 72a$

5. $a^2 + 5a - 24$

6. $r^2 + 2r - 48$

7. $x^2 + 6x - 72$

8. $d^2 + 2d + 80$

9. $4x^2 - 24x + 36$

10. $m^2 + 15m + 54$

11. $x^2 - 33x + 32$

12. $2x^4 - 24x^3 + 40x^2$

13. $b^2 + b - 72$

14. $d^2 - 25d + 156$

15. $b^4 - 14b^2 + 49$

16. $f^4 - 11f^2 - 26$

7.5 Notes

Name:______________________________________ Date:____________

Lesson 7.5: Factoring trinomials with a leading coefficient other than 1 Algebra I

For the last few days we have been factoring trinomials. So far we have only factored trinomials when the leading coefficient is equal to one. Today we will look at a method that will help us factor trinomials when the a value or leading coefficient is something other than 1.

First let's look at a multiplication example to help us understand why we will use this strategy.

$(2x-3)(5x+1)$

$2x\cdot 5x+2x\cdot 1-3\cdot 5x-3\cdot 1$

$10x^2+2x-15x-3$

$10x^2-13x-3$

Now in the last unit we looked at where the b and c numbers of our trinomial came from and we discovered that c came from the product the last numbers in the binomials and b came from the sum of the last numbers in the binomials. So let's look at the second line from the bottom of our work above. Look at the two middle terms. They are $+2x$ and $-15x$. This is interesting because their sum is in fact -13 but their product is -30. Where do you think that -30 could have come from? *Hopefully someone will figure out its* $10\cdot -3$! Meaning we should not list factor pairs for -3, but instead list factor pairs for $10\cdot -3$ or $a\cdot c$. But we are still looking for a sum of the original middle term.

Example problem: $2x^2 - 7x - 15$

Practice: Factor each trinomial

1. $2x^2+7x+6$

3. $2x^2+x-6$

2. $3x^2+2x-5$

4. $15+4x^2-17x$

5. $2+5x+2x^2$

9. $2x^2+3x-5$

6. $2x^2-5x-33$

10. $5x^2-9x+4$

7. $7x+3x^2+2$

11. $8x^2-18x+9$

8. $12x^2+11x-5$

12. $6x^2-x-2$

7.5 HW

Name:___________________________________ Date:____________

HW 7.5: Factoring trinomials $ax^2 + bx + c$ Algebra I

Factor:

1. $\mathbf{2x^2 - x - 10}$

2. $\mathbf{2x^2 - 3x + 1}$

3. $\mathbf{6x^2 - 4 + 5x}$

4. $\mathbf{3x^2 + 16x - 12}$

5. $\mathbf{3x^2 + 8x + 5}$

6. $\mathbf{27x + 4x^2 + 18}$

7. $\mathbf{20 + 12x^2 - 31x}$

8. $\mathbf{10x^2 + 9x - 9}$

Name____ Date_______________________

Lesson 7.6: Factor Completely Algebra I

Factoring a polynomial completely

1. Factor using GCF

2. Factor into 2 parentheses. You will either have:
   a. A perfect square to factor
   b. A trinomial to factor

****Your answer must have the GCF included before the parentheses!*

Example 1	$by^2 - 4b$	What was done?
Step 1:	$b(y^2 - 4)$	Factor out the GCF – it was *b*
Step 2:	$b(y + 2)(y - 2)$	Factored the perfect square. Notice the GCF is in front of the two parentheses.

Ex 2:	$4x^2 - 8x + 4$
Step 1: (GCF)	
Step 2: (Factor)	

Ex 3:	$x^3 - 4x$
Step 1: (GCF)	
Step 2: (Factor)	

Ex 4:	$4x^2 - 4x - 48$
Step 1: (GCF)	
Step 2: (Factor)	

Ex 5:	$5x^4 + 10x^2 + 5$
Step 1: (GCF)	
Step 2: (Factor)	

WHAT ABOUT THIS ONE?

Example 6	$x^4 - 16$	What was done?
Step 1:		There was no GCF
Step 2:	$(x^2 + 4)(x^2 - 4)$	Factored the perfect square.
Step 2 (again):	$(x^2 + 4)(x + 2)(x - 2)$	The second parenthesis is a perfect square. Factor the perfect square again.

Factor completely.

Ex 7: $2x^2 + 10x + 12$

Ex 8: $3x^3 - 15x^2 - 24x$

Ex 9: $3x^3 - 12x^2 - 63x$

Ex 10: $4x^2 + 28x - 120$

Ex 11: $2x^4 - 162$

Ex 12: $y^4 - 81$

Ex 13: $5x^3y - 20xy$

Ex 14: $x^6 - x^2$

Ex 15: $g^3 - g$

Ex 16: $ax^2 - 18ax + 77a$

Name____ Date________________________

HW 7.6: Factor Completely Algebra I

Factor each polynomial completely. Circle your final answer.

1. $4x^2 - 4$

2. $ax^2 - ay^2$

3. $st^2 - 9s$

4. $3x^2 - 27y^2$

5. $4a^2 - 36$

6. $y^4 - 81$

7. $3x^2 + 6x + 3$

8. $x^3 + 7x^2 + 10x$

9. $4x^2 - 8x + 4$

10. $y^4 - 13y^2 + 36$

Bonus

1. $25x^2 + 100xy + 100y^2$

Name____ Date________________________

Lesson 7.7: Factor Completely Algebra I

Factor completely. Circle your answer.

1. $2x^2 + 7x + 6$

2. $8x^2 + 20x - 24$

3. $-44 + 15x - x^2$

4. $x^2 + x = 30$

5. $3x^2 - 3y^2$

6. $-12x^2 - 8x + 20$

7. $6x^2 + 15x + 6$

8. $3x^4 - 48$

9. $-4x^2 + 26x - 30$

10. $4c^2 - 8c - 60$

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HW 7.7: Factor Completely Algebra I

1. $6x^2 + 11x + 3$

2. $4x^2 - 8x + 4$

3. $-3x^2 + 13x + 10$

4. $3ab^2 - 6a^2b$

5. $12x^4 - 27x^2 + 6$

6. $x^5 - 4x^4 - 21x^3$

7. $16y^2 - 64$

8. $4n^2 + 68n + 240$

9. $6x^2 - 6y^2$

10. $12y^2 + 3y - 9$

11. $4b^2 - 8b - 60$

12. $d^5 - 8d^3 + 16d$